# INTERCATEGORIAL ENTAILMENT

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*Abstract.* A notion of an intercategorial entailment, i.e. an entailment between expressions of different but functionally related categories, is formally defined. Some properties are indicated, and various examples are provided. They suggest that intercategorial entailment, although formally different from generalized entailment and from presupposition is a generalisation of both of these notions.

The usefulness in NL semantics of generalized cross-categorial entailment, i.e. entailment between two expressions of the same, but not necessarily sentential, category, is by now well-established (Keenan and Faltz 1985). Thus for any (major) category $C$, (possible) denotations of expressions of category $C$ form Boolean algebra $D_C$, and the entailment between expressions of category $C$ corresponds to the partial order proper to $D_C$. The tools introduced by Keenan have also been used to define a cross-categorial notion of presupposition (Zuber 1999). Thus it is possible to define a presupposition as a relation holding not only between sentences but also between expressions of other categories. In this article I define an entailment, the *intercategorial entailment*, or IC-entailment for short, between two expressions of different, but functionally related, categories.

Before providing a formal definition I give some examples involving IC-entailment at the pretheoretical level. Since all examples provided are basically heuristic it is preferable to ignore possible difficulties due to indexicality, plurality markers, tense interpretation, vagueness and other pragmatic and contextual factors.

The clearest examples of the IC-entailments are furnished by various complex nominals: in general their semantic content has a clear propositional character. Thus a definite description formed by a relative clause entails the sentence from which the relative clause is formed: the NP in (1a) entails, pretheoretically, the sentence in (1b):

(1a) The mosquito that Leo bought (1b) Leo bought a mosquito

Some specific NPs, the so-called exclusion and inclusion noun phrases (cf. Zuber 1998) also give rise to similar entailments: NPs in (2a) all entail the (declarative) sentence in (2b):

(2a) No student except Leo/every student except Leo and Lea/most students including Leo/no student, not even Leo
(2b) Leo is a student

A related series of examples concerns complex IC-entailing NPs which are parts of constituents of other categories: the NP in (3a) IC-entails (4) and the PP in (3b), the (full) VP in (3c) and the infinitival nominal in (3d) all pretheoretically entail (4):

(3a) the garden in which Leo kissed Lea
(3b) in the garden in which Leo kissed Lea
(3c) knows the garden in which Leo kissed Lea
(3d) to sleep in the garden in which Leo kissed Lea
(4) Leo kissed Lea in a garden

Similarly with the exclusion or inclusion NPs in (2a): when embedded syntactically they also preserve their IC-entailments: the VPs in (5a) and PPs in (5b) all entail (2b):

(5a) hates no student except Leo/every student except Leo and Lea/most students including Leo/no student, not even Leo
(5b) about no student except Leo/every student except Leo and Lea/most students including Leo/no student, not even Leo

An important class of expressions giving rise to IC-entailment is composed of the determiners from which exclusion or inclusion NPs are formed. Such determiners entail some specific NPs: all the determiners in (6a) entail the NP in (6b):

(6a) No...except Leo and Lea/Some...including Leo and Lea/most...including Leo and Lea
(6b) Leo and Lea

Similarily the determiners in (7a) entail the NP in (7b):

(7a) Every...except Leo and Lea/No..., not even Leo and Lea
(7b) Neither Leo nor Lea

Examples with possessive clauses and some nominalisations are similar: the NPs in (9a), (10a) and (11a) intuitively entail sentences (9b), (10b) and (11b) respectively:

(9a) Leo's bicycle
(9b) Leo has or had a bicycle (= Leo's bicycle exists)
(10a) Bill's ex-girl friend
(10b) Bill had a girl friend (=Bill's ex-girl friend exists)
(11a) Lea's eagerness to write with her left hand
(11b) Lea was or is eager to write with the left hand

Expressions of other categories can also give rise to IC-entailments. Absolute adjectives entail, when taken in their "literal" meaning, some specific common nouns; determiners may pretheoretically entail sentences, and transitive VPs may entail intransitive (full) VPs. The following examples illustrate these cases: (12a) entails (12b), (13a) entails (13b), (13b) entails (13c), (13a) also entails (13c), and (14a) entails (14b):

(12a) bold/pregnant
(12b) bold animal or human/ pregnant female or woman
(13a) No. . . except five/most. . . including at least five
(13b) Five beings/objects
(13c) There are at least five objects
(14a) ate and drank

(14b) ate something and drank something

It is also possible to have IC-entailments between expressions of a non-standard categories: the expression in (15a), admittedly of the category $S/VP$, entails sentence (15b):

(15a) Leo managed to (15b) Leo was trying to do something

Somewhat different examples involve non-declarative sentences. Interrogatives stand in entailment-like relation to sentences expressing their presuppositions, and exclamatives to the declaratives on which they are based (Zuber 1983). So, if we consider that interrogatives and exclamatives are of a non sentential category (or at least of a category different from the category of declarative sentences), then we have another example of IC-entailment. In particular the case of inclusion questions, as in (16a), is clear: the interrogative in (16a) entails the sentence in (16b). We also know that exclamative sentences intuitively entail (technically presuppose) the corresponding declarative sentences (Zuber 1983): the exclamative in (17a) entails the (declarative) sentence in (17b). Finally, imperative sentences also have presuppositions, which can be considered as a kind of IC-entailment: thus the imperative in (18a) presupposes the declarative sentence in (18b) and (18c):

(16a) Which student, in addition to Leo, went to the party?
(16b) Not only Leo went to the party
(17a) How beautiful Lea is !
(17b) Lea is beautiful.
(18a) Close the door and open the window !
(18b) The door is open and the window is closed
(18c) The window is closed

Examples with inclusive questions like the one in (16a) can also be used to show that IC-entailment is in principle different from presuppositions. For instance one can formally distinguish two IC-entailments of (16a) (Zuber 2000): the one in (19a) and the one in (19b):

(19a) Leo is a student (19b) Leo went to the party

It does not seem that both (19a) and (19b) express presuppositions of (16a).

Now I would like to give more theoretical arguments in favor of IC-entailment. The first argument, which was pointed out to me by Johan van Benthem (p.c.), is based on the observation that something very similar to IC-entailment, although categorially more restricted, in fact already exists in predicate logic or, for that matter, in most calculi with quantifiers or binding operators. Recall that in the classical predicate calculus we have two types of formulas between which logical relations hold: closed formulas and open formulas (formulas with free variables). In adition, open formulas are formed by expressions which may denote in different types. Thus, in general, we can have open formulas formed from n-ary relations, for various $n \in N$. But an i-ary relation does not denote the same type as a j-ary relation (if $i \neq j$). Given that in first order predicate logic the entailment is well-defined between closed and various open formulas, this just means that in classical predicate logic strictly speaking we also have an intercategorial entailment.

The second argument in favour of IC-entailment has to do with the existence in natural languages of functional expressions which are categorially polyvalent. These are expressions which are sometimes considered as categorially ambiguous since in different constructions they can take as arguments expressions of various categories. An important class of such expressions corresponds to those functional expressions which induce different categories on resulting expressions depending on the category of the argument. Here one can mention various polyvalent modifiers like for instance the focusing particles *only* or *also* (Zuber 2001). One of the most intuitive ways to explain the existence of categorially polyvalent items is to show that they may preserve some aspects of the meaning of their arguments, and that when they do so their arguments must stand in some specific logical or semantic relation. This is for instance the case with the examples in (20) where the verb *remembers/saw* takes different arguments and where there is a logical relation between (20a) and (20b):

(20a) Leo remembers/saw the car which killed the mosquito
(20b) Leo remembers/saw which car killed the mosquito

If one considers that it is not an accident but a regularity that the verbs in (20a) and in (20b) are semantically and lexically related, although strictly speaking they are of different categories, then IC-entailment may offer an explanation for this regularity. In other words, to make generalisations and predictions indicated in (20) we have to be able to compare "logically" expressions which are of different categories.

We are now in position to define formally the relation which corresponds to IC-entailment. I first indicate which categories are functionally related since the relation of IC-entailment holds between two expressions whose categories are functionally related. The syntactic framework assumed here is the framework of an extended categorial grammar which, in addition to the rule of function composition, has at least the rules of type raising and of function associativity. Functional expressions are expressions of the category $B/A$, and $B$ is called the *resulting category* (of the functional expression). From the semantic point of view I assume that the denotations of expressions of NLs form Boolean algebras which are atomic and complete. Let $D_C$ be the denotational algebra of expressions of category $C$. Then $D_{B/A}$ is called a functional (denotational) algebra and it corresponds to a set of functions from $D_A$ onto $D_B$. In general Boolean operations in $D_{B/A}$ are defined pointwise. So the notation $D_{B/A}$ is ambigous since it does not necessarily denote the set of *all* functions from $D_A$ onto $D_B$. This is because very often functions denoted by expressions of natural language are constrained in various ways. For instance $D_{Det}$, the algebra of possible denotations of one-place determiners, is a proper sub-algebra of the algebra of all functions from $D_{CN}$ onto $D_{NP}$: its elements are *conservative* functions from $D_{CN}$ onto $D_{NP}$. In practice it is obvious which denotational algebra is supposed to be represented by the notation $D_{B/A}$.

I will also make use of two sub-algebras of $D_{Det}$: these are algebras $INT$ of intersective functions and $CO-INT$ of co-intersective functions. By definition (Keenan 1993) $f \in INT$ (resp. $f \in CO-INT$) iff for all properties $X, Y, W, Z$, if $X \cap Y = W \cap Z$ (resp. $X - Y = W - Z$) then $f(X)(Y)$ is true iff $f(W)(Z)$ is true. Intersective functions are denoted by (generalized) existential determiners of the type *Some, No, No... except Leo*, etc. Similarly co-intersective functions are denoted by expressions of the type *Every, every... except Leo and Lea* etc.

Since some of the examples above involve IC-entailment between modifying functional expressions and their arguments, let me mention denotational algebras for modifiers. Modifiers are functional expressions of category $C/C$ for various choices of $C$. So there are modifiers of expressions of different categories. For every (major) category $C$, I will characterise two semantically different types of modifiers which give rise to IC-entailment. Modifiers of the first type, called *positive absolute*, denote in the sub-algebra $MOD_1(C)$ and, by definition $f_C \in MOD_1(C)$ iff $f_C \in D_{C/C}$ and for any $x \in D_C$, $f_C(x) = x \wedge f_C(1_{D_C})$. In the case of modification of common nouns, i.e. when $C = CN$, modifiers of this type correspond to absolute adjectives or to restrictive relative clauses. Modifiers of the second type denote in the sub-algebra $MOD_2(C)$. By definition (Zuber 1997) $f_C \in MOD_2(C)$ iff $f_C \in D_{C/C}$ and for any $x \in D_C$, $f_C(x) = x' \wedge f_C(0_{D_C})$. A trivial example of modifiers of this type corresponds to the negation of an expression of the category $C$ and which denotes the Boolean operation of complementation (in the corresponding $D_C$). Somewhat less trivial examples are furnished by complex modifiers formed from "negatively oriented items" such as *Nobody, Nowhere, Never*, etc. For instance the modifier *Nobody, not even...* is a NP modifier denoting a negative absolute function (and thus it corresponds to the type $MOD_2(NP)$). Similarly the modifiers *Never, not even at...* is a time adverbial modifier denoting a negative absolute function. Modifiers of this type are also those which are used in exclusion determiners or in negative inclusion determiners (Zuber 1998).

The idea underlying the definition of IC-entailment is simple: it reduces pointwise IC-entailment to the generalized entailment. As we have seen, it is possible to define Boolean algebras of functions from a set $A$ onto a Boolean algebra $B$ because operations for such functional algebras are defined pointwise by operations from $B$. So, if we have two expressions such that one of them is a functional expression and the other has the resulting category (of the functional expression) we can compare these two expressions pointwise at every value of the argument of the functional expression. This means that expressions between which IC-entailment holds are two expressions one of which is of category, say, $B/A$ and one of category $B$. Any two such expressions will be called *functionally related.*

I define the relation of IC-entailment not between expressions of an object language, but directly between expressions of the interpreting language or more precisely between elements of denotational algebras. Thus a simple definition of intercategorial entailment, $\leq_{IC}$, explicitly suggested by many of the above examples, is given in D1:

D 1: Let $S \in D_{C/A}$ and $T \in D_C$. Then $S$ intercategorially entails $T$, in notation $S \leq_{IC} T$ (resp. $T$ IC-entails $S$, $T \leq_{IC} S$) , iff for all $x \in D_A$ we have $S(x) \leq T$ (resp. $T \leq S(x)$).

Intuitively, according to D1, the inclusion of the semantic information involved in IC-entailment is guaranteed because this information is independent of the possible semantic values of arguments to which the functional expression applies. So, in some sense IC-entailments hold because possible semantic values of argument expressions do not contribute in any systematic way to this relation. All the examples given above clearly illustrate this point. We observe that all sentences having as the subject NP one of the NPs given in (2a) and as a VP any (grammatically) possible (extensional) VP whatsoever clearly entail the sentence in (2b). Examples in (3) also conform to the definition D1: in (3c) we have an VP which can be considered as a functional expression of the category $S/NP$. One can check that all sentences having as VP the VP in (3c) entail sentence (4).

To see that (3b) IC-entails (4) we need to use the rule of function associativity in the following form: any expression of category $(C/B)/A$ is also of the category $C/(B/A)$. Since an adverb is of category $VP/VP$ and a $VP$ is of category $S/NP$, by the function associativity an adverb is also of category $S/(NP/VP)$. So, roughly speaking, in this case an adverb can be considered as a functional expression which gives a sentence from a VP and a NP. Semantically such a function makes perfect sense. Now it is easy to verify that any sentence which has the VP modified by the adverb in (3b) entails the sentence (4). And this is precisely what definition D1 says.

Various properties which follow from D1 will probably clarify the intuition just indicated concerning the definition. First, note the following "contraposition law" which follows directly from D1:

Prop 1: Let $S \in D_{C/A}$ and $T \in D_C$. Then $S \leq_{IC} T$ iff $T' \leq_{IC} S'$ and $T \leq_{IC} S$ iff $S' \leq_{IC} T'$, where $S'$ and $T'$ are Boolean complements of $S$ and $T$ respectively.

Most of the examples given above illustrate IC-entailment as a relation between expressions of category $B/A$ and expressions of category $B$. By carefully choosing appropriate negations (Boolean complements) in these examples one can also show the validity of the IC-entailments in the other direction, in accordance with Prop 1. This can be done relatively easily with the examples given in (2). We want to show that (21a) IC-entails (21b), where *not no student except Leo* is the Boolean complement of *No student except Leo*:

(21a) Leo is not a student (21b) not (no student except Leo)

Notice now that if (21a) is true then the quantifier denoted by the exclusion NP *no student except Leo* is the empty set and consequently its Boolean complement contains all properties, which means that the function denoted by the NP in (21b) gives as value *truth* for any of its possible arguments. So (21a) IC-entails (21b).

The discussion of contraposition just given is useful to distinguish a new semantic relation which is a particular case of IC-entailment and which is related to presupposition by the duality principle. As we have seen in many of the cases discussed above, when the IC-entailed expression is a sentence then it corresponds to a presupposition. Indeed, we observe that at the sentence level the sentence of the form $NP\ VP$ presupposes the sentence $S$ if $NP$ IC-entails $S$. This means that $S$ is a presupposition of $NP\ VP$ if $NP\ VP$ entails $S$ and $NP - not\ VP$ entails $S$ (where $NP - not$ is the post-negation of $NP$). This corresponds to the classical deinition of presupposition in which presuppositional negation is the post-negation of the subject $NP$. Consider now the "contrapositions" of these two conditions of the classical definition of presupposition, as given in (22):

(22a) $not - S$ entails $not - NP\ VP$
(22b) $not - S$ entails $(not - NP) - not\ VP$

One can consider that (22a) and (22b) jointly define a new semantic relation which is the dual of presupposition (or a *post-supposition*): sentence $S$ dually presupposes sentence $T$ if $S$ entails $T$ and $S$ entails $not - T$. In order to get non-trivial post-suppositions we have to interpret $not - T$ as the sentence $T$ with the post-negation of its subject $NP$. For instance, given the above discussion, (21a) dually presupposes a sentence corresponding to (23):

(23) It is not true that every student except Leo is happy

One can find more natural cases of dual presuppositions by considering various conditional sentences (Zuber 2002).

I will now indicate other properties of IC-entailment and correlate them, where possible, with various examples given above. Since such a correlation is only possible for examples which have a well-known and generally accepted semantics examples involving non-declarative sentences will not be discussed in detail at this level. Prop 2 follows naturally from the definition D1 and Prop 1, given the associativity of function composition:

Prop 2: Let $S \in D_{(C/B)/A}$, $T \in D_{C/B}$ and $V \in D_C$. If $S \leq_{IC} T$ and $T \leq_{IC} V$ then $S \leq_{IC} V$.

The transitivity indicated in Prop 2 can be illustrated by examples given in (13): in (13a) we have a (nominal) determiner which IC-entails an NP in (13b) and which also IC-entails the sentence in (13c), the same sentence which is IC-entailed by (18a). We also have weaker transitivity, as in Prop 3:

Prop 3: If $S \leq_{IC} T$ and $T \leq V$ then $S \leq_{IC} V$ (where "$\leq$" is the ordinary generalized (cross-categorial) entailment.

Let us consider some sufficient conditions for IC-entailment. The first is related to monotonicity: since for any $x \in D_A$, $x \leq 1_{D_A}$ and $0_{D_A} \leq x$, we have:

Prop 4: If $f \in D_{B/A}$ is monotone increasing then $f \leq_{IC} f(1_{D_A})$ and $f(0_{D_A}) \leq_{IC} f$ and if $g \in D_{B/A}$ is monotone decreasing then $g \leq_{IC} g(0_{D_A})$ and $g(1_{D_A}) \leq_{IC} g$

Examples illustrating the first part of Prop 4 are given in (9) to (10) for instance: in (9a) and (10a) we have NPs which denote quantifiers which are monotone increasing functions (having as arguments the denotations of VPs). One notices that sentences obtained from (9a) and (10a) entail respectively (9b) and (10b), independently of the denotation of the VP (they cannot be intensional, however). Possibly the same is true of (11a) and (11b) but tense and intensionality have much greater influence here.

Proposition Prop 4 has a corollary concerning modifiers. Since functions denoted by modifiers of the first type (members of $MOD_1(C)$) are monotone increasing and modifiers of the second type (members of $MOD_2(C)$) are monotone decreasing, we have:

Prop 5: If $f \in MOD_1(C)$ then $f \leq_{IC} f(1_{D_C})$ and if $g \in MOD_2(C)$ then $g \leq_{IC} g(0_{D_C})$

The first part of Prop 5 can be illustrated by the examples in (12): in (12a) we have absolute adjectives, expressions of category $CN/CN$ which denote functions in $MOD_1(CN/CN)$ and in (12b) we have common nouns which denote the values of these functions at $1_{CN}$.

An important sufficient condition for the IC-entailment to hold is related to the atomicity of IC-entailing expressions, as the following property directly following from the definition of an atom, shows:

Prop 6: If $\alpha \in AT(D_{B/A})$ and $\beta = \bigvee_x \alpha(x)$ then $\alpha \leq_{IC} \beta$

The property in Prop 6 can be illustrated by examples in (6) and (7) above. More precisely in example (6a) the exclusion determiner denotes an atom of the $INT$, and in (7a) the (first) exclusion determiner denotes an atom of $CO - INT$. Furthermore the least upper bounds of the set of the values of these atoms are equal to filters or ideals, respectively, generated by the property determining the atoms.

The technique used to define IC-entailment can be used IC-consistency or IC-inconsistency.

For instance to define IC-inconsistency, one of the simplest ways would be to define it pointwise with respect to the functional expression: if $\alpha \in D_B$ and $\beta \in D_{B/A}$, then $\alpha$ and $\beta$ are IC-inconsistent iff for all $x \in D_A$ we have $\alpha \wedge \beta(x) = 0_{D_B}$. We have seen, however, given definition D2 and proposition Prop 4 that the relation of IC-entailment can hold not only between expressions of category $B/A$ and $B$. So it is possible to give a more general definition of IC-inconsistency: two expressions are IC-inconsistent iff they have incompatible IC-entailments. More precisely we have:

D 2: Let $\alpha \in D_{C/A}$ and $\beta \in D_{C/B}$. Then $\alpha$ and $\beta$ are IC-inconsistent iff for all $x \in D_A$ and all $y \in D_B$ we have $\alpha(x) \cap \beta(y) = 0_{D_C}$

As an example of IC-inconsistent expressions we can cite modifiers of type 2 (those denoting in $MOD_2(C)$): for any category $C$, and any $x \in D_C$ if $f_C \in MOD_2(C)$ then $f_C$ is IC-inconsistent with $f_C(x)$. Similarly, according to what has been said above about presuppositions of imperatives and of exclamatives, one should say that imperative sentences are IC-inconsistent with the corresponding exclamative sentences.

I gave definition D 2 and related examples just to indicate that the notion of IC-entailment should be related to other traditionally related notions. In fact, in more recent work in semantics, entailment is also related to Boolean binary operations - they are interdefinable and constitute general syntactic and semanic operations. Consequently the full picture of IC-entailment necessitates a separate but parallel developement on intercategorial conjunctions. This subjet matter is outside the scope of this introductory paper. I want to mention only that various ellipses can be considered as intercategorial conjunctions, two examples of which are given in (24), where a sentence is conjoined with an NP, under specific semantic conditions:

(24a) Leo did not call up, but only Lea (24b) Leo called up and only Leo

Since the application of the polyvalent modifier *only* to the modified expression results in an expression denoting an atom of the corresponding denotational algebra (Zuber 2001), examples in (24) suggest that IC-entailment may be useful to treat ellipses more directly and to make related generalizations concerning intercategorial conjunctions.

To conclude I would like to make some general remarks concerning the relation of IC-entailment to other "semantic containments" in order to ascertain its content better.

The first point is related to the definition of IC-entailment as such. First, from the strictly formal point of view the proposed definition does not have the most general form. It is possible to give a more general version of IC-entailment. In this case it holds between two expressions of different categories but which are in both cases Boolean categories. Thus, using the type-category notation, the expression denoting in type $(a_1 \rightarrow (a_2 \rightarrow \ldots(\ldots t)))$ may IC-entail the expression denoting in type $(b_1 \rightarrow (b_2 \rightarrow \ldots(\ldots t)))$. We have just to quantify out over all arguments in types $a_i, b_j$ and compare the end values in the type $t$ domain. Definition D2 is a particular case of this. Many properties of the IC-entailment, some of which were discussed above in connection with the simpler version, follow from such a generalized version. There are, however, not many non-trivial examples which could illustrate such a general definition of the IC-entailment.
The second remark is related to the observation that IC-entailment as defined above cannot hold between two expressions of arbitrarily different categories but only between expressions which have categories functionally related. This observation raises two related

questions: (1) How and in what sense is IC-entailment different from the "ordinary" cross-categorial entailment corresponding to the Boolean order proper to the given denotational algebra? (2) How can the relation of IC-entailment be extended to other categories, given the possibilities of multiple categorisation and type shifting?

The first question may seem strange since, as we have seen, from a purely empirical point of view, and given the traditional distinction of grammatical categories as provided by classical categorial grammar, expressions standing in the IC-relation are clearly of different categories, which is not the case with cross-categorial entailment.

From a purely technical point of view, however, one observes that denotations of expressions standing in the relation of IC-entailment are of the same algebraic nature, although of not of the same type. The reason is that elements of the algebra $D_B$ can also be considered as elements of the algebra $D_{B/A}$ (or a sub-algebra of this algebra) because they can be considered as constant functions from $D_A$ onto $D_B$: if $f \in D_B$ then there exists $g \in D_{B/A}$ such that for all $x \in D_A$ we have $g(x) = f$. Furthermore, since the $1_{D_{B/A}}$ and $0_{D_{B/A}}$ are constant functions, the set of all constant functions in $D_{B/A}$ forms a sub-algebra of $D_{B/A}$ which is isomorphic with $D_B$. Thus at some level IC-entailment is not very different from ordinary entailment. However, the presence of expressions denoting non-constant functions in the definition of IC-entailment makes it different from ordinary cross-categorial entailment. Notice in particular that if we try to define IC-equivalence in the usual way, i.e. as a conjunction of IC-entailments "in two directions", then we get IC-equivalence only between expressions denoting constant functions. But this is tantamount to saying that there is no proper IC-equivalence, as opposed to the ordinary cross-categorial equivalence. Notice also that given that functional expressions can denote constant functions, IC-entailment is a generalisation of ordinary cross-categorial entailment.

Concerning the second question above, IC-entailment and other related relations such as IC-inconsistency and IC-consistency, can hold between an expression and a whole family of expressions of different categories. This is guaranteed by the fact indicated in Prop 2 that IC-entailment is a "intercategorially transitive" relation. This means that IC-entailment is a generalization of the cross-categorial entailment. I have also tried to show, although in a somewhat speculative way, that IC-entailment does not cover all cases of the presupposed content (induced by some functional expressions). In that sense the IC-entailment does not correspond to the presupposition only.

Further inquiry leading to a more complete answer to the second question concerning the full variety of categories which can stand in the relation of the IC-entailment should distinguish two levels: the theoretical and the empirical. Theoretically, formal possibilities offered by the addition of various rules for multiple categorisation and category combination and extension should be analysed. Empirically, the possible influence of psychological or cognitive factors should be taken into account. Although formally IC-entailment is defined for an infinite number of functionally related categories, we have seen that some examples are more natural, and better illustrate the phenomenon than others. For instance the entailment from (complex) NPs to sentences seems much more grounded than entailment in other cases. Moreover, there are obviously categories which cannot stand in the relation of IC-entailment. This seems in particular to be the case of the NP on the one hand and of the VP on the other. Of course only deep empirical and theoretical inquiry may show that this is not an accident.

Concerning the definition of IC-entailment, whatever its version, the following complements should be added. Entailment is a semantic notion corresponding to a kind of inclusion in every model. My considerations and some technical results concern, strictly speaking, inclusion in one model. From the strictly logical point of view, interpreting functions need not be constrained in any particular way. It is known, however, that in natural language various restrictions on logically possible interpretations exist. Definition D1 uses basically the notion of cross-categorial entailment with which IC-entailment is defined. The cross-categorial entailment on which IC-entailment is based can be supposed, given the work in Keenan and Faltz (1985), to be not only formally correct but also empirically useful. In order to consider definition D1 formally correct, in addition to the usual syntactic specifications, we need to add various meaning postulates concerning universal constraints on interpreting functions. Such meaning postulates will stipulate that some functional expressions denote for instance monotone increasing functions in all models. The full specification of such constraints is a task which is still to be completed.

*Note*

An extended version of this paper was submitted in February 2000 to the *Journal of Logic, Language and Information*. It was refused for publication more than seventeen months later without any explanation from the editor in charge. One referee claimed that IC-entailment is a derivative notion and thus without any interest. By looking at her/his report I failed to be convinced that she/he read my paper carefully enough or that she/he had enough background to evaluate the paper properly. In the meantime the basic ideas of this paper have been presented on various occasions. In particular I gave a talk under the same title at Tokyo University in May 2000. I also presented a paper under the title "Intercategorial entailment and the semantic relations between non-declarative sentences" at conference "(Preferably) non-lexical semantics" (Paris, May 2000) and at the conference "Sinn und Bedeutung" (Amsterdam, December 2000). Finally I gave a talk with similar content at the Fifth Tbilisi Conference on Logic, Language and Computation in September 2001. I am indebted to various people who contributed directly or indirectly, volontarily or involontarily to the improvement, I believe, of this paper. So in particular I thank Johan van Benthem, Jeroen Groenendijk, Theo Janssen, Makoto Kanazawa, Markus Kracht, Daniel Lacombe and Ross Charnock.

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