Boolean Semantics and Categorial Polyvalency

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1. Introduction

Boolean semantics is a version of formal semantics in which models are supposed to have algebraic, preferably Boolean, structure (Keenan and Faltz 1985). Thus for any category C there is a corresponding denotational Boolean algebra D_C of possible denotations of expressions of category C. Furthermore, given that most categories are functionally related (in principle all "major" categories are Boolean), the corresponding denotational algebras are not independent of each other. In particular the algebra $D_{A/B}$ has as elements functions from D_B to D_A. Given that functions interpreting functional expressions may satisfy various "universal" constraints, we will usually consider just some sub-algebras of the algebra of all functions from D_B to D_A.

An important feature of denotational algebras D_C is that they are atomic. This means that any element different from the zero of the algebra contains an atom. An atom is an element which contains only itself and the zero element. Atoms of the algebra $D_{A/B}$ are determined by atoms and/or elements of the resulting algebra D_A. Obviously the binary algebra $\{0, 1\}$ has only one atom. This is the element corresponding to truth. Thus atoms of denotational algebras of functional expressions are eventually defined by truth.

Let me illustrate this point by the atomicity of $D_{A/B}$ in the case when there are no constraints on functions from D_A on D_B. In this case atomicity of $D_{A/B}$ is inherited from the atomicity of D_A. More precisely atoms of $D_{A/B}$ are determined by atoms of D_A in the following way (Zuber 2001):

Prop 1: For any $a \in D_B$, and for any atom $\alpha \in D_A$, the function $f_{a,\alpha}$ defined as $f_{a,\alpha}(x) = \alpha$ if $x = a$, and O_{D_B} otherwise, is an atom of $D_{B/A}$. Furthermore, every element of $D_{B/A}$ contains an atom of this form.

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 WCCFL 23 Proceedings, ed. B. Schmeiser, V. Chand, A. Kelleher and A. Rodriguez, pp. 829–832. Somerville, MA: Cascadilla Press.

Let us use this proposition to determine atoms of the denotational algebra NPs, a result which will be used in my analysis. Elements of D_{NP} are sets of sets. According to the above result, for any property P, the function f_P defined as $f_P(X) = 1$ if $X = P$ and $f_P(X) = O$ if $X \neq P$ is an atom of D_{NP}. Since such functions are characteristic functions of sets, atoms of D_{NP} are singletons containing a set as the unique element.

I will be interested in the semantics and thus in denotational algebras of a special class of modifiers. A modifier is a functional expression of category C/C for various choices of C. Thus by varying C we get, syntactically speaking, different modifiers. Modifiers of category C/C denote in the denotational algebra of restrictive functions $RESTR(C)$, which is a subset of the set of functions from D_C onto D_C. The set $RESTR(C)$ of restrictive functions $f_c \in D_{C/C}$, is the set of functions satisfying the condition $f_c(x) \leq x$, for any $x \in D_C$ (Keenan and Faltz 1985). The set of restrictive functions forms a Boolean algebra as is shown by the following proposition:

Prop 2: Let B be a Boolean algebra. Then the set of functions f from B onto B satisfying the condition $f(x) \leq x$ forms a Boolean algebra R_B with the Boolean operations of meet and join defined pointwise and where $0_{R_B} = 0_B$, $1_{R_B} = id_B$, $f'(x) = x \cap (f(x))'$.

Prop 2 shows how to form the restrictive Boolean algebra R_B from the algebra B. What is important here is the fact that the Boolean complement is relativised to the one element of the algebra which is just the identity function. If $B = D_C$ for a fixed category C, R_{D_C} will sometimes be denoted by $RESTR(C)$. This means that we have a family $RESTR(C)$, for any category C, of Boolean algebras formed as indicated in Prop 2. Restrictive algebras are also atomic (Keenan 1983):

Prop 3: If B is atomic so is R_B. For all $b \in B$ and all atoms α of B such that $\alpha \leq b$, functions $f_{b,\alpha}$ defined by $f_{b,\alpha}(x) = \alpha$ if $x = b$ and $f_{b,\alpha}(x) = 0_B$ if $x \neq b$ are the atoms.

There is an important sub-class $ABS(B)$ of restrictive functions (relative to a given Boolean algebra B): these are the so-called *absolute functions*. By definition $f \in ABS(B)$ iff for any $x \in B$, we have $f(x) = x \cap f(1_B)$. One can show (Keenan 1983) that $ABS(B)$ is a sub-algebra of R_B. The atoms and co-atoms of $ABS(B)$ are indicated in:

Prop 4: If B is atomic so is $ABS(B)$. For all atoms α of B, functions f_α, defined by $f_\alpha(x) = \alpha \cap x$ are the atoms of $ABS(B)$. For all atoms α of B, functions f_α, defined by $f_\alpha(x) = x \cap \alpha'$ are the co-atoms of $ABS(B)$

Observe that for any $A \in D_C$ the function $F(X) = X \cap A$ is an absolute function, member of $D_{C/C}$, and, moreover, all absolute functions can be represented as meets (conjunctions) in that way. Prop 4 indicates how atoms and co-atoms of $ABS(B)$ can be represented as conjunctions.

From the fact that $ABS(C)$ is a sub-algebra of $RESTR(C)$ it follows that absolute functions are restrictive ones. However, not all restrictive functions are absolute. In particular absolute functions are monotone increasing whereas restrictive non-absolute functions need not to be monotone. Furthermore, the algebra $ABS(C)$ is isomorphic to the algebra D_C.

The last denotational algebras that I will make use of are algebras of (some) determiners, that is expressions of category NP/CN: the algebra of intersective functions, INT, and the algebra of co-intersective functions, $CO - INT$ (Keenan 1993). By definition $F \in INT$, iff for all properties X, Y, Z and W, if $X \cap Y = Z \cap W$ then $F(X)(Y)$ is true iff $F(Z)(W)$ is true. Similarly, $F \in CO - INT$ iff for all properties X, Y, Z and W, if $X - Y = Z - W$ then $F(X)(Y)$ is true iff $F(Z)(W)$ is true.

Atoms of INT are functions at_P, where P is a property, such that $at_P(X)(Y)$ is true iff $X \cap Y = P$. Similarly atoms of $CO - INT$ are functions at_P such that $at_P(X)(Y)$ is true iff $X - Y = P$. For instance the exceptive determiner $No...except\ Leo$ denotes an atom of INT and the exceptive determiner $Every...except\ Leo$ denotes an atom of $CO - INT$.

Both INT and $CO - INT$ algebras are isomorphic to the algebra D_{NP} (Keenan 1993). This means that there is one-to-one correspondence between the bases of these algebras which preserves Boolean operations and which maps atoms of one algebra onto atoms of the other algebra.

2. Data

From the syntactic point of view the basic data to be considered here are constructions with the so-called categorially polyvalent particles (CPPs): these are (functional) expressions which can apply in a systematic way to expressions of different categories. This means that CPPs need to have multiple categorisation each of which accounts for the possible category of argument that a given CCP can take. The existence of expressions which need multiple categorisation was one of the motivations for an extension of categorial grammars (Keenan and Timberlake 1988).

The categorial polyvalency of focus items like *only, also* and *even* is well-known. Here are some examples:

(1) a. (Only/also/even Leo) danced on weekdays with Lea in the garden.

b. Leo (only/also/even danced) on weekdays with Lea in the garden.

c. Leo danced (only/also/even on weekdays) with Lea in the garden.

d. Leo danced on weekdays (only/also/even with Lea) in the garden.

e. Leo danced on weekdays with Lea (only/also/even in the garden).

f. Leo danced on weekdays with Lea in the garden.

One can see in these examples that *only/also/even* can apply syntactically to an NP, as in (1a); an intransitive verb or a verb phrase, as in (1b); an adverb, as in (1c); and different prepositional phrases, as in (1d) and (1e). These items can also modify numerals and conditional clauses (*only if/also if/even if*). Even if it is probably true that not all three are homogeneous in this respect, the above examples show that they are categorially polyvalent.

The following examples show that *especially*, *in particular* and *let alone* are also categorially polyvalent:

(2) a. Some teachers, in particular/especially Leo, think that ...

b. Yesterday he did many things, in particular he finished his paper.

c. He sings everywhere, in particular in his bathroom.

d. Leo will not come, let alone Lea.

e. Leo does not work on Saturdays, let alone on Sundays.

f. Leo does not smoke, let alone drink.

Surprisingly, *at least, at most*, which are usually considered to be numeral modifiers, are in fact also categorially polyvalent modifiers. This is shown by the following examples, which probably require in some cases a specific intonation in order to be fully acceptable:

(3) a. At least/at most Lea will pass the examination.

b. Lea sings at least/at most in the bathroom.

c. At least/at most five teachers were there.

We also observe that many CPPs can occur in conditional sentences.The cases where focus particles like *only, also* and *even* combine with *if*-clauses are well-known. In (4c) we have a modification of an *if*-clause by *at least* and (4d) shows that such a modification by *at most* is impossible:

(4) a. Lea will be happy, in particular if Leo calls.

b. Lea will not be happy if it rains, let alone if it snows.

c. Lea will call, at least if it rains.

d. *Lea will call, at most if it rains.

From the syntactic point of view CPPs represent a very heterogeneous group. Thus the classical focus particles like *only, also* and *even* and the particles *at least* and *at most* need not occur with additional lexical material when applying to a particular argument. This does not seem to be the case with particles like *especially* and *in particular*. Furthermore, there is a systematic semantic relationship between the additional lexical material and the argument of these particles suggesting that the explicitly required lexical material plays a role of an anaphora-antecedent like element:

(5) a. *In particular/especially Leo will call.

b. Some students, in particular Leo, will call.

c. Some students and in particular Leo, will call.

d. *He sings in his office, in particular/especially in the bathroom.

e. He sings everywhere, in particular in his bathroom.

f. He sings in his office and in particular in his bathroom.

g. *He likes wine, in particular chocolate.

h. He likes wine, in particular champagne.

i. He likes wine and in particular chocolate.

The sequence in (5a) is not acceptable because the anaphoric elements are lacking. In addition if the additional lexical material is present it must permit us to establish a particular relation of inclusion with the argument of the particle. Thus (5d) becomes acceptable if the bathroom is in the office. Similarly (5h) is acceptable whereas (5g) is not because champagne is a (kind of) wine but chocolate is not. The above examples show also that the necessary anaphoric element can be supplied in the form of a (first) conjunct making a conjunction of an appropriate category with the phrase modified by the particle. In this case the semantic relation of inclusion between the first and second conjunct is not necessary. We observe for instance that (5c), in contradistinction to (5b), does not entail that Leo is a student. Similarly (5i), where we have a conjunction, is acceptable, in contradistinction to (5g).

To conclude this section I want to make a few remarks concerning some pragmatic aspects of the above examples. Although it is difficult to avoid talking about a pragmatic contribution of every CPP, one can probably distinguish two classes of such particles in the examples above. So, roughly speaking, we have a class of particles whose general description does not necessitate a specific part devoted to their pragmatic contribution. This class includes essentially *only, also, at least* and *at most*.

The second class is formed from particles which induce an additional pragmatic or cognitive meaning aspect. The typical example here is *even*. Most analyses of this item assume that it makes three meaning contributions: assertoric, quantificational and pragmatic - which implies a surprise or unexpectedness that the object denoted by the argument of *even* has precisely the property indicated by the corresponding VP. Similar pragmatic effects expressing surprise, or its negation, can be associated with *especially* and *let alone*. What is, however, important and has not been stressed enough, is the fact that in all cases we have to do with a very similar pragmatic meaning contribution: in all cases which have been considered, writers speak about a surprise, an expectedness or a degree of likelihood (or their negation) of an event whose agent is, roughly speaking, in the scope of the particle. This poverty of possible pragmatic contribution is in sharp contrast to the great number of logical possibilities that lexical meanings from the field of cognition offers. In what follows I will try to offer an ”algebraic” explanation to this question as well.

3. Analysis

The main aim of this paper is to answer the question of how it is possible that CPPs keep their general meaning constant across categories. I propose to relate this meaning constancy of CPPs across categories by relating their denotations to atomicity of corresponding denotational algebras. Thus, in the simplest case an expression with a CCP denotes an atom in the algebra whose type is determined by the category of the argument of the particle. Other particles, and this is a more complex case, denote Boolean complexes, or Boolean combinations, of atoms and, possibly, of ”variables” of appropriate category. For instance expressions denoting co-atoms, that is Boolean complements of atoms, can also be considered as having a general, category independent meaning given that Boolean complements have such a meaning as well. Similarly a function of the form $f_c(x_c) = x_c \vee_c at_c$, can be considered as having a general meaning independent of category c because in its definition category independent operations are used.

There are some general arguments justifying this proposal. Atoms of Boolean algebras have a general meaning and can be defined independently of the type of elements of the algebra. In its definition one uses the notion of Boolean order (or its equivalent) and of the fixed elements of the algebra such as its zero element. These notions are also category independent. Furthermore, atoms are very specific elements. They are "minimal" elements and thus one can say that they are "exceptional". This feature of atoms will serve to explain the surprise effect of various particles: very roughly to denote an exceptional element is to surprise or to be unexpected. Finally, although atoms denoted by modifiers are not quantifiers, the definition of an atom non-trivially uses quantification. This fact may explain the quantificational effect associated with many particles (*only, also, even, especially*, etc.).

I will now illustrate my proposal with some examples. Since I am rather interested by the general idea underlying my proposal I will not analyse all of the particles specified above. Neither will I try to make a full description of a given individual particle that I will analyse.

Let me first look at the classical focus particles *only, also* and *even*. We observe that all these particles are semantically modifiers denoting restrictive functions. This means in particular that the sentences with a particle entail the corresponding "particle-less" sentence; in particular all sentences in (1) entail (2). Furthermore, one can also say that these particles keep their meaning constant independently of the category of expressions to which they apply. This meaning constancy in my proposal is due to the fact that their denotations are linked to atomicity. So let us look at these particles from this angle.

The case of *only* is relatively easy. We can explain its meaning constancy across categories by saying that *only* always denotes atoms of the denotational algebras of modifiers (Zuber 2001). Which exact atom and in which algebra depends on the category and value of the argument of *only*. Thus *only* in *only NP* denotes an atom in $D_{NP/NP}$, *only* in *only yesterday* denotes an atom in $D_{VP/VP}$, *only* in *only five* denotes an atom in the denotational algebra of modifiers of numerals (or determiners), etc. Of course this does not mean that for arguments of some specific categories one does not encounter some problems, as is the case of VPs for example. Such problems are more general, however, and independent of the treatment of specific particles.

This proposal concerning the relationship between *only* and atomicity can be justified more easily for some categories than for others. One can give an "almost formal" proof (Zuber 2001) that *only NP* denotes an atom of D_{NP} using the fact that there is an isomorphism between the algebra D_{INT} of intersective determiners and the algebra D_{NP} . The mapping establishing the isomorphism associates with the $D \in D_{INT}$ the $Q \in D_{NP}$ in such a way that $Q = D(E)$ (cf.Keenan 1993), where the E is the universal property (the universe). Now, as we have seen, the determiner *No* ... *except Leo* denotes an atom in the algebra of intersective functions. The above isomorphism associates with this atom the element $NO(OBJECT)\ EXCEPT\ LEO$, the denotation of *Nobody except Leo* (=*only Leo*). Since isomorphisms match atoms to atoms we have the needed result.

This reasoning based on the isomorphism in conjunction with the isomorphism between $ABS(NP)$ and D_{NP} suggests that *only* when applied to NPs denotes an atom of $ABS(NP)$ and not an atom of $RESTR(NP)$. This is compatible with proposition 4 concerning atoms of $ABS(C)$.

Let us consider now the particle *also*. One of the simplest ways to represent this particle would be to analyse it as the Boolean complement of *only* as indicated in (6):

(6) $ALSO(X) = ONLY'(X)$

Since, as indicated above, co-atoms also have general type independent meaning this suggestion would be in agreement with the general proposal I make here. It does not seem, however, that this solution is satisfactory. For indeed, it seems that the quantificational value of *also* excludes the universal quantifier strength as shown by the contrast in the following examples;

(7) a. ?Leo and also everybody danced.

b. *Not only Leo but also everybody danced.

c. Leo and even everybody danced.

d. Not only Leo but even everybody danced.

So if the constraints suggested by examples in (7) are real, the semantics of *also* given in (8) would be more appropriate;

(8) $ALSO(X) = ONLY'(X)\ AND\ (EVERYTHING)'$

It is important to note that the second conjunct in (8) can also be expressed in "type independent language": $EVERYTHING$ is the intersection of all principal filters (in D_{NP} if $X = NP$). So we can define the element corresponding to $EVERYTHING$ in all other denotational algebras.

The representation in (8) does not seem to solve all the problems. In particular, the representation in (8) (or the one in (6)) does not account for the presuppositional differences between *only* and *also*, that is the fact that, roughly, what sentences with *only* presuppose seems to be asserted by the corresponding sentences with *also*. Of course to solve this difficulty more should be said about presupposition in the context of atomic expressions. It seems indeed that clear cases of presupposition relation usually involve atomicity. Somewhat anticipating a full discussion this topic, left to another occasion, a representation like the one in (9) would be more appropriate than the one in (8):

(9) $ALSO(X) = ONLY'(not - X)$

For reasons of formal correctness it should be specified what the negation $not - X$ refers to and at the same time what the algebraic structure of the argument X is. I will not do this here.

In order to analyse *even* it is useful to look at some constraints concerning possible occurrences of this item:

(10) a. *Some/*most/*five teachers, even Leo, are dancing.

b. Every teacher, even Leo, is dancing.

c. No teacher, not even Leo, is dancing.

d. Some/most/five teachers, and even Leo, are dancing.

As (10) shows the grammaticality of some complex determiners of the form *DET...even NP* depends on the type of the determiner *DET*. Apparently only the universal determiner *every* (and negative universal *no* in negative sentences) can occur on the position of *DET*. In (10d) we have an acceptable sequence because we have a conjunction of NPs.

The above examples should be compared with the following:

(11) a. *Some/*most/*five teachers except Leo are (not) dancing.

b. Every/no teacher except Leo, is dancing.

It is generally assumed that exceptive determiners as in (11) also obey a determiner constraint: it seems that complex determiners of the form *DET...except NP* admit on the position of *(*DET) only universal or negatively universal quantifier (Moltmann 1995, Zuber 1998). Although many counter-examples have been found to this claim (cf. Garcia-Alvarez 2004) and although the agrammaticality of examples in (10) is subtle, the above examples give us a hint as to how analyse *even*. We have seen that exceptive determiners denote atoms of the algebra INT or $CO - INT$. This means in particular that (11b) is true iff the following holds: $\{T - D\} = \{L\}$. Since there is a relationship between co-intersective determiners and NPs this leads to the following analysis of *even* modifying an NP: (12a) has its semantics informally given in (12b), and more formally in (12c):

(12) a. Even Leo danced.

b. There is a (non-atomic, i.e. different from a singleton) property which no dancer but Leo has.

c. For some P, $D \cap P = \{L\}$

Thus *even*, when applied to an NP, indicates that this NP is the only NP to have a specific property among all those who have the property indicated by VP of the sentence in which the NP modified by *even* occurs (in the subject position). Moreover, a this is the pragmatic aspect of the story, this specific property is contextually considered as incompatible with the property of being a dancer, expressed by the VP, making thus Leo an ”implausible dancer”.

Notice that it follows from these remarks that *even* can be analysed as denoting an atomic restrictive (non-absolute) function. Thus it follows from (12b) or (12c) that no other object than Leo has a property corresponding to the meet $D \cap P$: non-dancers may be ”implausible dancers” and all dancers but Leo are ”plausible” dancers

The fact that properties are atoms of (denotations of) NPs allows us to generalise the analysis in (12) to other categories to which *even* can apply. Suppose ”$S($*even*$(E_C))$” designates a sentence in which the constituent E_C of category C is in the scope of *even*. Then, roughly, if this sentence is true there exists an atom $\alpha \in D_C$ such that among all possible constituents which can replace E_C *salva veritate* in this sentence only the constituent E_C contains α.

More complex Boolean compounds are denotations of other categorially polyvalent particles. For instance the cross-categorial semantics of *at least* is given in (13):

(13) $AT\ LEAST(X) = X\ OR\ NOT - ONLY(X)$

Thus, according to (13), *at least in the bathroom* is equivalent to *in the bathroom or not only in the bathroom* and *at least if it rains* is equivalent to *if it rains or not-only if it rains*.

Concerning *at most* we have at least two solutions: they depend on whether this particle induces a specific pragmatic content or not. For the cases when there is no such content, like for instance when numerals are modified, the semantics of *at most* is given in (14) (where $NOTHING$ is the meet of all principal ideals, so can be defined cross-categorially):

(14) $AT\ MOST(X) = ONLY(X)\ OR\ NOTHING$

The pragmatic content induced by *at most* may express a modalised surprise; thus (15a) can be read with the content given in (15b):

(15) a. At most Leo danced.

b. If someone danced then it should not be surprising that it was Leo.

As we have seen such a pragmatic content is related to *even*, which is interpreted by the atom of restrictive non-absolute modifiers. So in such cases *at most* should be interpreted along lines indicated in (16):

(16) $AT\ MOST(X) = ONLY(X)\ OR\ NOT - EVEN(X)$

Thus *at least* and *at most* can be expressed as Boolean combinations of atoms and categorised variables in a way which makes their representations type independent as well. Interestingly enough such complex modifiers do not denote restrictive functions anymore: *AT MOST(X)* does not entail *X*.

4. Conclusions

The basic question I wanted to answer in this paper is how categorially polyvalent particles, which syntactically are modifiers, can keep constant their abstract meaning across categories. I propose that the particles in question denote atoms or elements defined with the help of Boolean operations and atoms. The exact type of these atoms depends on the category of the argument of the particle: if the argument is of category C the denotation of the particle will be related to the atom of the denotational algebra $D_{C/C}$. Since atoms have their specific meanings independently of the type of the denotational algebra atoms of which they are instances, expressions related to atoms keep their meaning constant across categories, in the same way as atoms do independently of which algebra they are atoms of.

This resort to the algebraic notion of an atom has two other advantages. It explains the quantificational effect which many CPPs induce even though, strictly speaking, they are not quantifier denoting expressions. This quantificational effect is also related to the definition of an atom, since such a definition implicitly makes use of quantification in a non-trivial way.

The second advantage of the proposal concerns the pragmatic aspects of the meaning contribution of some CPPs particles. The resort to atomicity may also explain the surprise effect of *even* and, the negation of surprise in the case of *especially/in particular*. I suggest that these items should be analysed with the help of atoms of restrictive (non-absolute) functions as defined in Prop 3. From this point of view my proposal is close to the suggestions of Karttunen and Peters (1979) that categorially polyvalent particles, at least *even*, do not contribute to the truth-conditions. Indeed we notice that when *even* applies to an NP the most natural case is one in which the NP in its scope denotes a principal filter (for instance proper nouns or conjunctions of proper nouns denote filters). But then the condition specifying the ”meaning” of *even* given in (12b) and (12c) is logically trivial: it corresponds just to a property any filter has. So in that sense it does not add any new semantic (logical) information. However, the situation is different if we assume that this condition is related to atomicity. Such algebraic information allows us to understand some cognitive effects (unexpectedness) and also in some cases the quantificational effect. Although more detailed development is in need, we can say that the surprise effect comes from the uniqueness and specificity of atoms distinguished by categorially polyvalent particles. For instance in (12a) the particle *even* indicates that Leo has a property which no other dancer has. This property is contextually considered as incompatible with dancing and consequently leads to the unexpectedness effect.

The explanation I offer for the semantic contribution does not suppose any total ordering of elements with respect to the unexpectedness effect, in opposition to some more pragmatically oriented explanations which suppose scales with total ordering (Fauconnier 1975). For this reason it is compatible with the data in (17):

(17) a. Even Leo and Lea danced.

b. *Even Leo and even Lea danced.

c. All teachers, especially Leo and Lea are happy.

d. *All teachers, especially Leo and especially Lea are happy.

These facts are clearly compatible with the proposal based on atomicity since a conjunction of two atoms leads to a contradiction.

Recall that often the strong pragmatic contribution of specific particles has been at the basis of criticisms of truth-conditional semantics, or of the semantic nature of presupposition, given that it has usually been associated with the presupposed content (Karttunen and Peters 1979, Wilson 1975). I suggest, somewhat speculatively, that by relating pragmatic presuppositions, or more generally a pragmatic content, induced by certain particles, it might be useful to include some algebraic tools to the usual machinery of truth-conditional semantics, partly to get rid of the often fuzzy pragmatic terminology and machinery. In particular I suggest an extension of the usual logical language used in semantic representations by some algebraic concepts (related to atoms in particular).

I want to conclude with two general remarks. I illustrated my proposal with a few examples all of which involve modifiers which denote restrictive functions. Natural languages contain many other categorially polyvalent modifiers which do not denote restrictive functions but, on the contrary, so to speak, negatively restricting functions. As shown in Morzycki (2003) items like *almost, hardly, nearly,* etc. belong to this category of modifiers. It may also happen that there are some abstract categorially polyvalent operations, which are not lexically marked. This may be the case with the question operator (cf. Zuber 2000). Obviously there are no formal difficulty with treating these phenomena along the lines suggested here. There are various Boolean algebras corresponding to these new possibilities which are also atomic (cf. Zuber 1997, 2003) and consequently my proposal can be easily carried out with these items as well.

My last remark concerns the cases of categorially polyvalent items which do not keep the same meaning but change it in some way across categories or sub-categories. As a possible example of such an item one can mention *again* or *already*. These items may vary their meaning according to the specific category of expressions to which they apply or even according to the type, tense or aspect of the verb which they modify. Similarly the French particle *même* can mean *even, (him)-self* or *(the) same* according to the category of its argument or even its position with respect to the argument. Of course we need first to make clearer the notion of meaning constancy or meaning variation in this context. I want to emphasise, however, that my approach does not exclude the possibility that some particles need not keep their meaning constant. This is due to the fact that although an atom has a general meaning defined with the help of the partial order, strictly speaking in this case we talk about atoms of a particular algebra. So, in particular, elements which are atoms in one algebra are not atoms of its sub-algebra and atoms of a sub-algebra are not atoms of the corresponding algebra. This means, supposing that a sub-categorisation of grammatical categories corresponds to taking sub-algebras of denotational algebras, that meaning variation of categorially polyvalent particles is possible. For the same reason in a given language there may be many particles which have similar meanings (as for instance *also, too, as well* in English) and there may exist cross-linguistic differences concerning these particles.

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