

Where do Presuppositions Come from ?

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1 Introduction

A rich body of data supplied on the phenomenon of presupposition leads to the following empirical generalisation: overwhelmingly presuppositions, at least semantic presuppositions, attached to a particular sentences have as their source a specific expression occurring in that sentence. Furthermore, one notice that such expressions, *presupposition inducers* or *presuppositions triggers* act "locally" in the sense that they need not to have a whole sentence as argument and that the semantic content of the presupposition to which they give rise is usually independent of the semantic content of many other expressions occurring in the presupposing sentence. Thus, to take the classical example, the presupposition of existence and possibly of uniqueness associated with sentences containing noun phrases with the definite *the* (on subject position) is independent of the content of the verb phrase of the presupposing sentence.

One can talk about inducers of various semantic contents. Clearly we can speak about entailment inducers. For instance we can say that the propositional function *It is true that* induces entailment because *It is true that P* entails *P*, for any *P*. Similarly we can say that conjunctions are (cross-categorial) entailment inducers and (extensional) adjectives induce cross-categorial entailment between common nouns: *ADJ CN* entails *CN*, for any common noun *CN*. It is important, however, that these entailment inducers are not presupposition inducers: *It is true that P* does not presuppose *P* and *A and B* does not presuppose *A*.

The purpose of this paper is to characterise semantically presupposition inducers, in particular in their opposition to entailment inducers. I will show that there are two related types of presupposition inducers, those which roughly induce lexical presuppositions and those which induce non-lexical (or functional) presuppositions. As we will see those presupposition inducers are not necessary inducers of presuppositions expressed by sentences. In fact the relation of presupposition will hold between expressions of any grammatical category.

In this paper presupposition inducers will be identified with a class of inter-categorially entailing expressions. Intercategorial entailment holds between two expressions which are of different but functionally related categories. Thus a functional expression *F* of category *A/B* inter-categorially entails an expression *G* of category *A* if *F(X)* (cross-categorially) entails *G* for any *X* of category *B*.

To define inducers of lexical presuppositions we will need a notion of relativised Boolean algebras: these are algebras in which the unit and possibly the zero element have been changed, under suitable conditions. To see usefulness

of this move let me recall the known difficulty with defining the presupposition with the help of negation, understood as the Boolean complement. I indicate this difficulty in the general case when the relation of presupposition holds between two expressions which are not necessarily sentences. Suppose we say that E_1 presupposes E_2 iff E_1 and E_1' , the Boolean complement of E_1 , both entail E_2 . Since the only element in which an element and its Boolean complement can be included is the unit element of the corresponding Boolean algebra this means that the only presuppositions we obtain by this definition are trivial elements, necessarily true sentences in the case of sentential presuppositions. To avoid this difficulty I propose to use Boolean algebras in which the unit element has been changed: given a Boolean algebra B and its element b we can form a Boolean algebra $B(b)$ whose base is the set $\{x : x \leq b\}$. This set constitutes a new algebra $B(b)$, where b is the unit element and where the complements are elements relativised to b : for any $x \in B(b)$ we define $c(x)$, the Boolean complement of x in $B(b)$ as: $c(x) = x' \wedge b$, where x' is the Boolean complement of x in B .

It is easy to see that we can define the relation of presupposition, at least for some type of presupposition, as we will see, using relativised algebras: this is because given the relativised algebra $B(b)$ it is true that any element $x \in B(b)$ and its complement $c(x)$ both are included in, or entail b . Since the element b needs not to be trivial we obtain that way non-trivial presuppositions via a version of the classical definition. This is the main idea. We need to show how the element b playing the role of the unit element is to be chosen in a non arbitrary way and how this idea can be implemented to account for the known cases of presupposition at sentential and non-sentential levels. In particular more has to be said as to what kind of elements, formally speaking, can be said to presuppose. The paper is organised as follows. In the next section I present some formal tools which will be used in the paper. This presentation will be completed in each section to come by a material directly related to the semantics of specific constructions discussed in various sections. Then in next sections I discuss the case of some focus particles, of presupposing quantifiers and of factive constructions. This discussion is given for purely illustrative reasons and should not be considered as constituting a description of some presupposition inducers in English. Some general remarks conclude the paper.

2 Some formal preliminaries

In this section I recall some basic notions of Boolean semantics (cf. Keenan and Faltz 1985). More specific formal tools will be presented in next sections when discussing some specific constructions giving rise to presuppositions.

Boolean framework for semantics means, briefly, the following. Every expression of a given natural language is associated with at least one grammatical category. With every category C is associated its denotational algebra D_C , which is a set of possible denotations of expressions of category C . Denotational algebras D_C are atomic (and complete) Boolean algebras. The partial order in denotational algebras is interpreted as a generalized entailment. Thus it is mean-

ingful to say that an entailment holds between two NPs, between two nominal determiners, between two VPs, etc. In particular it is meaningful to say that expressions (of a given category) denoting atoms entail other expressions (of the same category). An entailment which holds between two expressions of the same, not necessarily sentential category, is called *cross-categorial* entailment.

Given a Boolean algebra B one can associate with it two classes of (related) algebras. First, we have the class of relativised algebras related to B . Thus for any $a, b \in B$ such that $a \leq b$, the set $B(a, b) = \{x : a \leq x \leq b\}$ forms a Boolean algebra where the meet and join are the same as those in B , the zero element equals to a , the unit element equals to b and the complement is relativised to a and b . More precisely for any $x \in B(a, b)$, the complement x' of x in $B(a, b)$ is defined as $x' = (c(x) \wedge b) \vee a$, where $c(x)$ is the complement of x in B .

The second way class of algebras corresponds to a set of functions which have the given algebra as the range. More specifically let B be a Boolean algebra and A an arbitrary set. Then the set $F(A, B)$ of all functions from A into B forms a Boolean algebra in which Boolean operations are defined pointwise. For instance the meet \wedge_F in this algebra is defined as follows: for any $f, g \in F(A, B)$ and any $x \in A$, $(f \wedge_F g)(x) = f(x) \wedge_B g(x)$, where \wedge_B is the meet in B . Similarly with other operations. The zero element 0_F of $F(A, B)$ is the constant function from A into B which takes always 0_B as its value: $0_F(x) = 0_B$ for any $x \in A$.

The notion of a functional Boolean algebra allows us to talk meaningfully about *inter-categorial* entailment (IC-entailment) as well. The IC-entailment holds between two expressions which need not be of the same category but whose categories are functionally related (Zuber 2002). Two expressions are functionally related iff their category index terminates in the same category. Two category indices terminate in the same category iff either they are both of the same category or else their categories are respectively of the form C/α and C/β for any, possibly empty, category α and β . If α is the empty category then the category C/α equals to C . Thus all Boolean categories are functionally related since all of them terminate in S . Given the associativity of the operation of category formation two expressions E and F which are functionally related can be explicitly represented with their categorial indices as: $F_{C/A}$ and $G_{C/B}$ where A, B, C are categories and A, B are possibly empty categories. Given this notation we define the IC-entailment as follows (cf. Zuber 2002):

D1: Expression E of category C/A IC-entails expression F of category C/B , if and only if for all expressions x of category A (possible arguments of E) and all y of category B (possible arguments of F), the expression $E_{C/A}(x)$ cross-categorially entails the expression $F_{C/B}(y)$.

According to D1, an IC-entailment is a pointwise defined cross-categorial entailment. Given the definition of functionally related categories, the classical entailment between sentences and cross-entailment between expressions of the same category are particular cases of IC-entailment.

Let us see some examples of IC-entailments. It is easy to see now that NPs in (1a) IC-entail the sentence in (1b):

- (1a) No student except Leo/Most students including Leo
 (1b) Leo is a student

This is because any sentence obtained from the NP in (1a) by adding to it any (extensional) VP whatsoever entails the sentence in (1b). Similarly we observe that, when used with subject NP, the determiners in (2a), which are of category $NP/CN = (S/VP)/CN$, IC-entail the sentence in (2b):

- (2a) No...except five/All..., including five.
 (2b) There are at least five objects.

Among various properties of the IC-entailment we retain the following: if a functional expression of the category A/B denotes a monotonic increasing function F then this expression IC-entails the expression denoting $F(1_{D_A})$.

Examples in (1) and (2) suggest that IC-entailment is directly related to presupposition (cf. Zuber 1997): it is very tempting to say that (1a) (inter-categorially) presupposes (1b) and that (2a) presupposes (2b). To spell it in more details we need in addition some notions concerning atomicity of denotational algebras.

As indicated, denotational algebras, in particular functional and relativised algebras are atomic. In general the atomicity of $D_{B/A}$ is inherited from the atomicity of D_B . When there are no constraints on functions from D_A on D_B , atoms of $D_{B/A}$ are determined by atoms of D_B in the following way (Zuber 2001):

Fact 1: For any $a \in D_A$, and for any atom in D_B , the function $f_{a,\alpha} \in D_{B/A}$ defined as $f_{a,\alpha}(x) = \alpha$ if $x = a$, and O_{D_B} otherwise, is an atom of $D_{B/A}$. Furthermore, every element of $D_{B/A}$ contains an atom of this form.

Let us use this proposition to determine atoms of the denotational algebra of NPs, a result which will be used in the sequel. Elements of D_{NP} are sets of sets. According to the above result, for any property P , the function f_P defined as $f_P(X) = 1$ if $X = P$ and $f_P(X) = O$ if $X \neq P$ is an atom of D_{NP} . Since such functions are characteristic functions of sets, atoms of D_{NP} are singletons containing a set as the unique element.

Some other, more general, properties of atoms will be discussed in the next section where more atomic algebras will be introduced.

3 Semantics of presupposition inducers

Many papers on presuppositions mention various "presupposing constructions" which in spite of a great syntactic variety seem to exemplify, at least pre-theoretically, the same phenomenon (Zuber 1972, Soames 1989, Beaver 1997).

Whether this is indeed the case and in what precise sense is out of point of this paper. I am trying to characterise without any pretention of exhaustivity but in as general as possible way basic semantic aspects of these presupposing constructions and more particularly of those expressions which can directly be considered as presupposition inducers. More specifically I will characterise some specific quantifier expressions giving rise to presuppositions, in particular exclusion and exclusion phrases, some focus particles, in particular expressions like *only* and *also*, and, finally the factive predicates.

Syntactic observations concerning presuppositions inducers indicate that they are functional expressions. This means that preferably they should be analysed as expressions of the category A/B . Very often, in particular in the case of focus particle they are modifiers, that is expressions of category A/A . Thus presupposition inducers denote in functional algebras. Furthermore, given the general discussion in the introduction these algebras should be relativised algebras. This means that we need to define relativised functional algebras. We will consider relativised algebras in which only the unit element has changed.

Let $D_{A/B}$ be a functional algebra and a an element of A . Then:

Fact 2: The set $F(a, A/B)$ of functions from D_B onto D_A such that $F(a, A/B) = \{f : f \in D_{A/B}, f(x) \leq a\}$ for any $x \in B$ forms a Boolean algebra with the Boolean operations defined as follows:

- (i) the zero element, the meet and the join operations in $F(a, A/B)$ are the same as in $D_{A/B}$
- (ii) the unit element in $F(a, A/B)$ is the constant function 1_F such that $1_F(x) = a$ for any $x \in D_B$.
- (iii) the complement $c(f)$ of f in $F(a, A/B)$ is defined as $c(f) = f' \cap 1_F$, where f' is the complement of f in $D_{A/B}$

We will call algebras as presented in fact 2 *functional relativised algebras*. The unit element of such algebras is the relativising element.

Relativised algebras are atomic. More precisely we have:

Fact 3: If the algebra $D_{A/B}$ is atomic then the relativised algebra $F(a, A/B)$ is also atomic. The function $f_{\alpha,b}$ such that $f_{\alpha,b}(x) = \alpha$ if $x = b$ and $f_{\alpha,b}(x) = 0$ otherwise are atoms of $F(a, A/B)$ (where α is an atom of D_A such that $\alpha \leq a$ and $b \in B$). Furthermore, any atom of $F(a, A/B)$ is of that form.

A particular case of functional relativised algebras are algebras for modifiers. A modifier is a functional expression of category C/C for various choices of C . Thus by varying C we get, syntactically speaking, different modifiers.

Various modifiers can denote in various denotational algebras. Keenan and Faltz (1985) distinguish two such algebras: the algebra *RESTR* of *restrictive functions* and its proper sub-algebra *ABS*, represented by the set of absolute functions. The set $RESTR(C)$ of restrictive functions $f_c \in D_{C/C}$, is the set of functions satisfying the condition $f_c \leq id_c$ (where $id_c(x) = x$, for any $x \in$

D_C). Thus $RESRT(C)$ is the functional relativised algebra with the relativising element equal to identity function: $RESTR(C) = F(id, C/C)$.

Restrictive algebras are also atomic:

Prop 1: If B is atomic so is R_B . For all $b \in B$ and all atoms α of B such that $\alpha \leq b$, functions $f_{b,\alpha}$ defined by $f_{b,\alpha}(x) = \alpha$ if $x = b$ and $f_{b,\alpha}(x) = 0_B$ if $x \neq b$ are the atoms.

The algebra $ABS(B)$ of absolute functions is a sub-algebra of $RESTR(B)$: $f \in ABS(B)$ iff for any $x \in B$, we have $f(x) = x \cap f(1_B)$. The atoms and co-atoms of $ABS(B)$ are indicated in:

Prop 2: If B is atomic so is $ABS(B)$. For all atoms α of B , functions f_α , defined by $f_\alpha(x) = \alpha \cap x$ are the atoms of $ABS(B)$. For all atoms α of B , functions $f_{\alpha'}$, defined by $f_{\alpha'}(x) = x \cap \alpha'$ are the co-atoms of $ABS(B)$.

We can say now more about atoms. We notice that atoms and co-atoms denoted by functional expressions of category C/D are determined by, or are "indexed", by the atoms of the resulting denotational algebra D_C . Since in the case of modifiers the category of the resulting expression is the same as the category of the argument, the index of atomic and co-atomic absolute restrictive functions, elements of D_C/C , is an atom of D_C . Furthermore in natural languages atomic modifiers are strongly syncategorematic in the sense that their atomic index is always logically related to the (denotation of) the argument (Zuber 2004a). Usually this relationship corresponds to the generalized entailment: the index of the atomic function denoted by a modifier in a given modified expression entails (is included in) the denotation of the argument. Thus the atomic function denoted by *Only in Only Plato* is different from the atomic function denoted by *Only in Only Socrates and Plato*. Indeed in the former case this function is indexed by the singleton $\{P\}$, where P is the referent of *Plato*, and in the latter case the atomic function is indexed by the two element set $\{P, S\}$. Similarly with the modifier *exactly* considered as modifier of numerals denoting in the algebra *CARD*: *exactly in exactly n* denotes an atomic function (member of the algebra of absolute functions from *CARD* into itself) which is different from the function denoted by *exactly in exactly m* (for $m \neq n$). This is because the former function is indexed by n and the latter by m .

The syncategorematicity of atomic and co-atomic expressions is an the basis of my proposal concerning the presupposition inducers. I propose the following preliminary definition:

D2: A functional expression E is a presupposition inducer only if:
 Either E is an atomic or co-atomic modifier (for non-lexical presuppositions)
 or E denotes a member of a relativised functional algebra

D2 has the following consequences: (1) if a presupposition inducer I is of category A/B then presupposition holds between expressions of the category A , which needs not be a sentential. (2) presuppositions are IC-entailments of presupposition inducers. They are related to expressions indexing atoms (in the case of non-lexical presuppositions).

Of course given above we need rules for the calculation of presuppositions of sentences from non-sentential presuppositions and components. One such rule which will be illustrated in next sections is the following: if a functional expression E_1 presupposes functional expression E_2 then $E_1(A)$ presupposes $E_2(A)$.

In next sections I present various illustrations of definition D2.

4 Presuppositional quantifiers

We know that some quantified NPs induce presuppositions and other do not. For instance NPs corresponding to definite descriptions give rise to specific presuppositions whereas universally quantified NPs do not have such a property. In order to show how my proposal accounts for this type of differences we need some tools from the GQT which I will assume to be essentially known and only in part in need to recall (for more details see Keenan and Westerstahl 1997).

GQT deals with the semantics of NPs and some of their syntactic parts, determiners. Denotations of NPs are (generalized) quantifiers of type $\langle 1 \rangle$. They are thus, as we have seen, functions from sets to truth-values. Quantifiers of type $\langle 1, 1 \rangle$ are denotations of (unary) determiners, i.e. expressions of category NP/CN and members of D_{Det} . It is assumed that D_{Det} properly contains an atomic sub-algebra $CONS$ of conservative functions. One distinguish sometimes a special class of non-conservative members of D_{Det} : these are functions obtained from conservative functions by the operation of argument inversion (Zuber 2005).

Quantifiers of type $\langle 1 \rangle$ and of type $\langle 1, 1 \rangle$ have two type of complements: the usual Boolean complement and the *post-complement*. The post-complement of the type $\langle 1 \rangle$ quantifier Q , noted $Q-n$, is defined as: $Q-n = \{Y : Y' \in Q\}$.

There are two sub-algebras of $CONS$ (cf. Keenan 1993): the atomic algebra of intersective functions, INT (which includes denotations of such determiners as *some*, *no*, *no... except five*, *no... except Leo*), and the atomic algebra of co-intersective functions, $CO-INT$ (which includes denotations of determiners like *every*, *every... but Plato*, *every... except ten*. Atoms of INT are functions at_P , where P is a property, such that $at_P(X)(Y)$ is true iff $X \cap Y = P$. Similarly atoms of $CO-INT$ are functions at_P such that $at_P(X)(Y)$ is true iff $X - Y = P$.

The algebra INT has a sub-algebra $CARD$ of cardinal functions. Atoms of this algebra are determined by a cardinal: for any cardinal α the function f_α such that $f_\alpha(X)(Y) = 1$ iff $|X \cap Y| = \alpha$ is an atom of $CARD$. Numerals and the exceptive expressions of the form *No... except n* denote cardinal functions. The algebra $CO-CARD$ is a sub-algebra of $CO-INT$. For any cardinal α the function f_α such that $f_\alpha(X)(Y) = 1$ iff $|X - Y| = \alpha$ is an atom of $CO-CARD$. The exceptive expressions of the form *Every... except n* denote co-cardinal functions.

I conclude this presentation by the algebra *GCARD* of generalised cardinals (cf. Zuber 2005). By definition $F \in GCARD$ iff for all properties X, Y_1, Y_2 if $|X \cap Y_1| = |X \cap Y_2|$ then $F(X)(Y_1) = F(X)(Y_2)$. *GCARD* is atomic: for any set A and any cardinal n the function $F_{A,n}$ such that $F_{A,n}(X) = 1$ iff $X = A$ and $|X \cap Y| = n$ and $|X| = n$ is an atom of *GCARD*. Clearly *CARD* and *CO-CARD* are sub-algebras of *GCARD*. Thus numerals and exceptive expressions like *No...except n*, *Every...except n* denote in *GCARD*.

I have insisted on atomicity of various algebras for determiners because atomic expressions induce presuppositions. Here are some examples:

- (3) Every/no student except Leo is a vegetarian.
- (4) Apart from Leo only vegetarians are students.
- (5) Leo is a student.

We want to say that (3) presupposes (5) and that (4) presupposes the (post) negation of (5). Let us see how we get the first case.

Notice first that the subject NPs of (3) inter categorially entail (5). Furthermore these NPs are of the form *Det CN* and the *Det* denote atomic function. The determiner *Every...except Leo* denotes an atom of *CO-INT* and the determiner *No...except Leo* denotes an atom of *INT*. These atoms are determined by *Leo* and they are results of application of the (atomic) determiners modifier *Every...except* or *No...except* to the determiner *the... who is Leo* (because *Every student except Leo* means *Every student except the student who is Leo*). In other words the atomic determiner *Every except/no except* chooses, semantically speaking, an atom from its argument. According to my proposal this means that the determiner *Every ...except Leo* presupposes *the/every...who is Leo*. Furthermore, this entails, given that the expressions which stand in the relation of presupposition are functional expressions, that *Every student except Leo* presupposes *Every/the student who is Leo*. This last NP denotes a monotonic function and consequently IC-entails *The student who is Leo exists* which is equivalent to *Leo is a student*.

To deal with determiners like the one in (4) we have to observe that *Apart from Leo...* denotes an atom of the algebra of functions which are inverses of co-intersective functions (cf. Zuber 2005).

A special case of presupposing quantifying expressions is constituted by expressions with inclusion determiners that is determiners like *some/most/every/ten...including Leo*: (6) also presupposes (5):

- (6) Some/most/all students, including Leo, are vegetarians.

We account for these facts by noting that inclusion determiners of the above type determine co-atomic functional expressions which they entail.

Finally we account for the classical example of presuppositions induced by *the* by observing that *the* can be considered as an atomic modifier of elements denoting in *GCARD*: for any n the determiner *the n* denotes an atom in *GCARD*.

5 *Only* and *also*

The presupposition inducers *only* and *also*, and their variants, have played historically an important role (cf. Horn 1969) and their presuppositional description is very instructive. It is thus usually assumed that (9a) presupposes (9b) and asserts (9c). Concerning *also* it is usually said that (10) presupposes (11a) and asserts (11b):

- (9a) Only Leo is a philosopher.
- (9b) Leo is a philosopher.
- (9c) Nobody who is not Leo is philosopher.
- (10) Leo also is a philosopher.
- (11a) Someone who is not Leo is a philosopher.
- (11b) Leo is a philosopher.

We observe also that that presuppositions and assertions induced by *only* and *also* are related: the presupposition of *only* is an assertion of *also* and the assertion of *only* corresponds to a kind of negation of the presupposition of *also*.

Furthermore, we observe that the post-negation does not preserve the presupposition in the above cases. Thus (12a) presupposes (12b), and not (9b) and (13a) presupposes (13b), and not (11a);

- (12a) Only Leo is not a philosopher.
- (12b) Leo is not a philosopher.
- (13a) Leo also is not a philosopher.
- (13b) Someone other than Leo is not a philosopher.

Similarly the NP *only Leo* entails (cross-categorially) *Leo* but does not IC-entail neither (9b) nor (10b). It is also true that the post-negation of (11a) does not entail (11b).

Finally we observe that *only* and *also* are NP modifiers in the above examples and thus they denote restrictive functions. Moreover, as we have already seen, *only* denotes an atomic restrictive function. Concerning *also* there are some arguments to consider that it denotes the complement of *ONLY* (cf. Zuber 1998, 2004). We notice in particular that *not only* cross-categorially entails *also*.

The above observations are sufficient to show how my proposal works in the case of *only* and *also*. Even if it is not quite clear whether *only* denotes an atom of restrictive intersective or non-intersective functions we can suppose that this function is determined by the argument of *only* (possibly by an atom of the denotation of the argument). Thus *only* in *only Leo* denotes the atomic function F such that $F = ONLY_{\{L\}}$, if F is intersective or $F = ONLY_{L, \{L\}}$ if F is non-intersective. It follows from this that *only Leo* presupposes *Leo* because the following is true for any $X \in D_{NP}$: $F(X) \leq L$.

By a similar reasoning we conclude that *Leo also* presupposes *someone else* ($\neq Leo$) because *also* indexed by *Leo* inter-categorially entails *someone else* (since $F'(X) \leq X$ for any $X \neq L$ and if $X = L$ then $F'(L) \leq SOME-NON-L$).

Notice finally that my proposal accounts for the fact that *only* and *also* are categorially polyvalent presupposition inducers because the notion of an atom or co-atom is type-independent (cf. Zuber 2004b).

6 Factives

Factives are probably first non-quantificational and non-nominal expressions which have been said to induce presuppositions. One considers, after the work Kiparsky and Kiparsky 1967, that there are, roughly speaking, two types of verbs which can take sentential complements: those which entail their sentential complement and those which do not. Thus (17) and (18) entail (20) whereas (19) does not entail it:

- (17) Leo knows that joking is useful.
- (18) Leo regrets that joking is useful.
- (19) Leo believes that joking is useful.
- (20) Joking is useful.

One important observation done by Kiparskys is that not only (17) and (18) entail (20) but also that their "natural" negation entails it and thus that there is a relation of presupposition between (18) and (19) on the one hand and their complement sentence on the other. Another observation they made concerns the distinction, which they tried to justify solely on syntactic grounds, between emotive and non-emotive factives. Thus emotive factives are verbs and predicates like *to regret*, *it is sad*, *it is surprising*, etc. whereas non-emotive are represented by *to know (that)* or *to remember (that)*.

The distinction between emotive and non-emotive factives is more deep than suspected and can be done on semantic grounds. It has been observed (Zuber 1977) that emotive factives, roughly, entail, or even presuppose, knowledge whereas non-emotive factives assert it. Thus it seems very plausible to assume not only that (18) entails (17) and that (21) entails (22) but also that the (verbal) negation of (18) entails (17) and the (verbal) negation of (21) entails (22):

- (21) It is fun/sad/surprising that joking is useful
- (22) It is known that joking is fun

This observation leads directly to the way in which emotive factives should be analysed. We have to suppose that emotive factives denote in a relativised Boolean algebra which has knowledge as the unit element.

Concerning non-emotive factives the following solution is possible. Some observations suggest (cf. Zuber 1977) that, roughly speaking, the full sentence taking predicates formed from non-emotive factives entail (presuppose) knowledge "by some-one": *Leo knows that* and *Leo does not know that* entail *Someone knows that*. The reason is that negations entailing the "universal ignorance" are

not easy as the following well-known examples show:

(23) ?No-one/nobody knows that joking is useful.

(24)? I do not know that joking is useful.

(25) *It is not (well) known that joking is useful.

Thus, according to this suggestion knowledge, as expressed in natural languages, is related to a particular knowing agent. This has as consequence the fact that presupposition inducers corresponding to non-emotive factives are expressions like *Leo knows that, Lea, but not Leo knows that, everybody but Leo knows that, Leo does not know that*, etc. These expressions denote in the relativised denotational algebra which has as the unit element the denotation of *Someone knows/It is known that*.

I need now to make some comments about lexically different than *to know* possible non-emotive factives. As mentioned above verbs like *forget, remember, realise* are usually considered as belonging to that class. One observes that these verbs are related to knowledge and it is in general possible to "define" them by the verb *to know* and an other presupposition inducer. Thus, roughly speaking, we have: *forget=not to know anymore, remember=still know*, etc. This means that the class of non-emotive factives is determined by different possible subjects of the verb *to know* and not by different verbs.

7 Concluding remarks

Extending some proposals from Zuber (1997) and Zuber (1999) I suggested that presuppositions are intercategoryal consequences of specific functional expressions, presupposition inducers. There are two reasons for which presupposition inducers intercategoryally entail presuppositions. First, they may denote in functional relativised algebras and thus intercategoryally entail the unit elements of such algebras giving rise to lexical presuppositions. Second, they give rise to intercategoryal entailments by virtue of the fact that they denote atoms or co-atoms of specific denotational algebras determined by the entailed expression. In this case we have non-lexical presuppositions.

My proposal has the following consequences: (1) Basically expressions which presuppose are not sentences. Consequently presupposition projection exists at sub-sentential level as well. The framework of Boolean semantics allows for a natural calculation of presuppositions of complex expressions from presuppositions of their parts. (2) Presuppositions have peculiar behaviour in the context of various "conjunctions" because Boolean operations with atoms and unit elements give special results. (3) Presuppositions can "vanish" because presupposition inducers may cease to denote atomic functions in some contexts. This is because atoms of a sub-algebra need not be atoms of the corresponding super-algebra. (4) Classical logic is not sufficient to define presuppositions because in the algebra $\{0, 1\}$ the value 1 coincides with the only atom of this algebra and this algebra does not have any (non-trivial) relativised algebra. (5) Presuppositions

have metalinguistic flavour because they are due to "algebraic necessity" (properties of atoms and unit elements in an algebra). (6) My proposal is compatible with various "dynamic" approaches because the change of the unit element in relativised algebras may be considered, at least in some cases, as a dynamic process.

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