Reflexive and Reciprocal Determiners

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Abstract. Reflexive (*most..., including himself* and reciprocal (*no... except each other*) determiners are anaphoric determiners. They form arguments of transitive verbs which cannot occur in subject position of sentences. Various logical properties (invariance, conservativity, a-conservativity, a-intersectivity) of functions denoted by these determiners are studied. These properties account for their anaphoricity and show formal differences between anaphoric and ordinary determiners.

1 Introduction

According to the well-established terminology, ("ordinary") determiners are functional expressions which take one or more common nouns (CNs) as arguments and give a noun phrase, (NP), as result. For instance *every, most, five, no except two* and *more... than...* are determiners. Syntactically NPs are arguments of intransitive, transitive or ditransitive verb phrases (VPs), that is they can occur as subjects, direct or indirect objects. There are, however, expressions which are arguments of verbs but which cannot occur in all argumental position of the verb, and thus, which are not, strictly speaking NPs:

(1) a. Leo and Lea kissed each other.
    b. * Each other kissed Leo and Lea.

(2) a. Leo and Lea washed themselves.
    b. *(They)selves washed Leo and Lea.

The reciprocal *each other* is an argument of the verb *kiss* in (1) where it occurs as a direct object. As shown in (1b) this reciprocal cannot occur in the subject position. Similarly, the reflexive *themselves* occurs in the object position in (2a) but it does not have the (corresponding) nominative form which could occur in the subject position.

Reciprocals and reflexives belong to the class of generalised NPs (GNPs) that is these nominal expression which typically fulfil the function of arguments of the main clause and thus can serve as arguments of (transitive) VPs. Obviously "ordinary" NPs are also GNPs. However, reciprocals and reflexives are proper (genuine) GNPs because, contrary to "ordinary" NPs, proper GNPs cannot occur in all argumental positions of a transitive VP, in particular they cannot occur in the subject positions, not even in the subject positions of simple

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intransitive sentences. Typical examples of such GNPs are the reflexive pronouns *himself/herself/themselves* and the reciprocal pronoun *each other*. These can be Booleanly combined with other GNPs, proper or "ordinary", to give complex GNPs such as *each other but not themselves, himself and most students, ten students including each other and themselves*, etc. Here are some examples of sentences containing Booleanly complex GNPs:

(3) a. Leo admires himself and most linguists.
b. * Himself and most linguists admire Leo.

(4) a. Leo and Lea admire themselves and each other.
b. *Themselves and each other admire Leo and Lea.

In this paper I do not study GNPs in general, even if some differences between ordinary NPs and genuine GNPs will be indicated in Sect. 4. I will study here, in a preliminary way, functional expressions forming some GNPs. Functional expressions forming ordinary NPs, that is *(nominal) determiners* forming a DP (or a NP) from a CN have been extensively studied. Formal properties of (full) reciprocals and reflexives are studied in Zuber (2016). In this paper I analyse formal properties of (1) *reflexive determiners* (RefDets) that is functional expressions which take a CN as argument and form a reflexive GNP (like for instance *most..., including himself and Lea*) and (2) reciprocal determiners (RecDets), that is functional expressions which take a CN as argument and give a reciprocal GNP as result (like for instance *no... except each other and themselves*). Both these classes of functional expressions form generalised determiners (GDets). In addition, as will be shown below, GNPs formed by GDets considered here are anaphors. In that sense they are different from other GDets forming GNPs such as *the same* or *a different number of*, which do not form anaphors when applied to a CN.

I will be specifically interested in logical and semantic properties of functions denoted by RefDets and by RecDets. These properties will indicate formal similarities and differences between "ordinary" determiners (those forming "ordinary" DPs with a CN) and GDets considered here. They will also indicate differences and similarities between reflexives and reciprocals. Two kinds of such properties wiil be discussed: those related to the anaphoricity of reflexive and reciprocal determiners and those related to the conservativity of functions they denote. Concerning conservativity, two, logically related, types of it will be discussed, one of which is characteristic for anaphoric determiners.

In the next section we indicate in some detail the data we will be concerned with. Then formal tools from the extended Generalised Quantifier Theory are recalled. In Sect. 4 the semantics of various RefDets and RecDets is provided and Sect. 5 discusses formal properties which show differences and similarities between functions denoted by anaphoric determiners and quantifiers denoted by ordinary determiners.

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2 Some Data

RefDets and RecDetds have been only scarcely discussed even if much more have been written about RefDets. Both these classes can be divided into possessive and non-possessive GDets. Some (but not all) languages have "marked" or morphologically simple possessive RefDets. The possessive anaphoric pronoun SVOJ in Slavic languages (as opposed to EGO) or *hans* in Norwegian (both meaning roughly *his/her own*) are probably well-known (Zuber 2009). The Polish pronoun *swój* can in addition combine with virtually any other "ordinary" determiner to give a series of complex possessive RefDets which in English corresponds to the series like *all of his own, most of his own, ten of his own*, etc.

Concerning possessive RecDets we have the possessive form *each other's* and various Boolean combination of it with "ordinary" (non anaphoric) possessives determiners. Thus *each other's but not Bill's ..., everybody's, including each other's...* are possessive RecDets as in the following examples:

(5) a. Leo and Lea help each other's but not Bill's (students).
   b. Leo and Lea help each other's and their own (students).

Interestingly, in Polish, the possessive RefDet SVOJ can, in many situations have the meaning corresponding to possessive RecDet *each other's*.

Non-possessive RefDets and RecDets are obtained from specific "ordinary" determiners. One can distinguish two classes of such RefDets: those obtained from, roughly speaking, inclusive determiners, and those obtained from exclusive determiners (Zuber 1998, Zuber 2010b). Inclusive determiners are discontinuous determiners of the form *Det,..., including EXP* (where *Det* is an ordinary simple determiner denoting a monotone increasing (on the second argument) type $\langle 1, 1\rangle$ quantifier) and exclusive determiners are determiners of the form *every/no... except EXP* The expression *EXP* is the complement of *including* or of *except*. Both these classes of determiners form a NP when applied to a CN.

By replacing the complement *EXP* of *including* by an expression which denotes a **PI** function (see below) we get inclusive anaphoric RefDets. Thus inclusive anaphoric RefDets are expressions of the form *Det, including himself/herself* or of the form *Det, including NP and himself/herself.* For example the following expressions are RefDets: *most...including herself, most...including some Albanians and himself, ten...including herself and two Japanese*, etc. The last determiner occurs in (6a). Observe that (6a) means (6b) and apparently cannot mean (6c). This fact is related to the anaphoricity of the determiner involved in (6a):

(6) a. Lea admires ten students, including herself and two Japanese.
   b. Lea admires ten students including herself and two Japanese students.
   c. Lea admires ten students including herself and two Japanese which are not students.

There are also "negative" inclusive determiners from which we can obtain Ref Dets and RecDets. In the following examples *no..., not even himself* is such a RefDet and *no... not even each other* is such a RecDet:

(7) a. Leo admires no linguist, not even himself.
b. Leo and Leo admire no linguist, not even each other

We will not analyse here such "negative" inclusive anaphoric determiners.

Exclusive ordinary determiners are determiners such as *every...except Leo, every... but two, no...except Japanese, no...except Albanian and Sue*, etc. By replacing in them the complement of *except* by a reflexive GNP (that is an expression whose denotation satisfies **PI** and does not satisfy **EC**) we can form RefDets like the following: *every... except himself, no...except Leo and herself.* The following sentence contains such a RefDet:

(8) Leo and Lea hate every linguist except themselves.

Not surprizingly, non-possessive RecDets can also be formed from the inclusive and exclusive "ordinary" determiners by putting as the complement of *including* or of *except* a reciprocal GNP. Thus in (9a) and (9b) we have RecDets based on inclusive "ordinary" determiners and in (10a), (10b) and (10c) - RecDets based on exclusive determiners:

(9) a. Leo and Lea hate most vegetarians, including each other.
b. Most teachers admire some Japanese, including each other and themselves.

(10) a. Leo and Lea admire no philosopher except each other and Plato.
b. Three linguists admire every linguist except each other.
c. Two monks admire no philosopher, except each other and themselves.

This way of constructing non-possessive RefDets and RecDets from the ordinary inclusive and exclusive determiners is productive in many languages.

Let us see now some differences between possessive and non-possessive anaphoric determiners in their relation to the class of "ordinary" determiners. Up to now we have considered only unary determiners. Natural languages have also n-ary determiners (Keenan and Moss 1985). For instance (11a) can naturally mean what (11b) means in which case *most...and...* should be considered as binary determiner. In other words the admiration of Leo concerns two groups of people: a group of linguists and a group of philosophers. Similarly in (12) we have a binary determiner *more...than...*:

(11) a. Leo admires most linguists and philosophers.
b. Leo admires most linguists and most philosophers.

(12) Lea knows more linguists than philosophers.

One observes that possessive RefDets and RecDets can take many CNs as arguments as seen in (13) and (14):

(13) Leo burnt more of his own paintings than letters.

(14) Leo and Bill like each other's books and articles.

It does not seem that there are non-possessive RefDets or non-possessive RecDets taking many nominal arguments: in (15) and in (16) only one group of people is involved, those who are linguists and philosophers "at the same time":

(15) Leo and Lea admire most linguists and philosophers, including themselves.

(16) Leo and Lea admire all linguists and philosophers, except each other.

In addition to *except* and *including* some other connectors can be used to form non-possessive anaphoric determiners. This is the case with *apart from* and, possibly, *in addition to.* Constructions with such connectors will be ignored in what follows.

In the next section we give the semantics for various types of anaphoric determiners presented above. Even if it is possible to extend various definitions given in the preceeding section to n-ary determiners, we will consider only the semantics of unary determiners. Furthermore, we will not provide the semantic description of possessive anaphoric determiners. Semantic properties of some possessive determiners are discussed in Zuber (2009).

3 Formal Preliminaries

We will consider binary relations and functions over a universe E, assumed to be finite throughout this paper. $D(R)$ denotes the domain of the relation R. The relation I is the identity relation: $I = \{\langle x, y\rangle : x = y\}$. If R is a binary relation and X a set then $R/X = R \cap (X \times X)$. The binary relation R^S is the greatest symmetric part of the relation R, that is $R^S = R \cap R^{-1}$. A symmetric relation R is cross-product iff $R = A \times A$ or $R = (A \times A) \cap I'$ for some $A \subseteq E$. If R is a symmetric relation then $\Pi(R)$ is the least fine partition of R such that every of its blocks is a cross-product relation and every two blocks have incompatible domain: if $B_1 \in \Pi(R)$ and $B_2 \in \Pi(R)$ then $D(B_1) \cap D(B_2) = \emptyset$. A partition is 1. *atomic* iff every of its blocks is a singleton; 2. *simgular* iff it contains only one block (which is not a singleton); 3. *non-trivial* iff it is neither atomic nor singular.

If a function takes only a binary relation as argument, its type is noted $\langle 2 : \tau\rangle$, where τ is the type of the output; if a function takes a set and a binary relation as arguments, its type is noted $\langle 1, 2 : \tau\rangle$. If $\tau = 1$ then the output of the function is a set of individuals and thus its type is $\langle 2 : 1\rangle$ or $\langle 1, 2 : 1\rangle$. The function $SELF$, denoted by the reflexive *himself* defined as $SELF(R) = \{x : \langle x, x\rangle \in R\}$, is of type $\langle 2 : 1\rangle$ and the function denoted by the anaphoric determiner *every...but himself* is of type $\langle 1, 2 : 1\rangle$. We will consider here also the case when

τ corresponds to a set of type $\langle 1\rangle$ quantifiers and thus τ equals, in Montagovian notation, $\langle\langle\langle e,t\rangle t\rangle t\rangle$. The type of such functions will be noted either $\langle 2:\langle 1\rangle\rangle$ - functions from binary relations to sets of type $\langle 1\rangle$ quantifiers)) or $\langle 1,2:\langle 1\rangle\rangle$ - functions from sets and binary relations to sets of type $\langle 1\rangle$ quantifiers.

Basic type $\langle 1\rangle$ quantifiers are functions from sets to truth-values. In this case they are denotations of subject NPs. However, NPs can also occur in the direct object positions and in this case their denotations do not take sets (denotations of VPs) as arguments but denotations of TVPs (relations) as arguments. To account for this eventuality one extends the domain of application of basic type $\langle 1\rangle$ quantifiers so that they apply to n-ary relations and have as output an (n–1)-ary relation. Since we are basically interested in binary relations, the domain of application of basic type $\langle 1\rangle$ quantifiers will be extended by adding to their domain the set of binary relations. When a quantifier Q acts as a "direct object" we get its *accusative case extension* Q_{acc} (Keenan and Westerstahl 1997):

Definition 1. *For each type $\langle 1\rangle$ quantifier Q, $Q_{acc}R=\{a:Q(aR)=1\}$, where $aR=\{y:\langle a,y\rangle\in R\}$.*

A type $\langle 1\rangle$ quantifier Q is *positive* iff $Q(\emptyset)=0$ and Q is *atomic* iff it contains exactly one element, that is if $Q=\{A\}$ for some $A\subseteq E$. We will call a type $\langle 1\rangle$ quantifier Q *natural* iff either Q is positive and $E\in Q$ or Q is not positive and $E\notin Q$; Q is plural, $Q\in PL$, iff if $X\in Q$ then $|X|\geq 2$.

A special class of type $\langle 1\rangle$ quantifiers is formed by *individuals*: I_a is an individual (generated by $a\in E$) iff $I_a=\{X:a\in X\}$. More generally, $Ft(A)$, the *(principal) filter generated by a set.A*, is defined as $Ft(A)=\{X:X\subseteq E\wedge A\subseteq X\}$. Principal filters generated by singletons are called *ultrafilters*. Thus individuals are ultrafilters. They are denotations of proper names. NPs of the form *Every CN* denote principal filters generated by the denotation of *CN*. Meets of two principal filters are principal filters: $Ft(A)\cap Ft(B)=Ft(A\cup B)$. Thus conjunctions (supposed to denote meets) of proper names denote principal filters generated by the union of referents of the proper names.

We will use also the property of *living on* (cf. Barwise and Cooper 1981). The basic type $\langle 1\rangle$ quantifier lives on a set A (where $A\subseteq E$) iff for all $X\subseteq E$, $Q(X)=Q(X\cap A)$. If E is finite then there is always a smallest set on which a quantifier Q lives. If A is a set on which Q lives we will write $Li(Q,A)$ and the smallest set on which Q lives will be noted $SLi(Q)$.

A related notion is the notion of a witness set of the quantifier Q, relative to the set A on which Q lives:

Definition 2. $W\in Wt_Q(A)$ *iff* $W\in Q\wedge W\subseteq A\wedge Li(Q,A)$.

Thus $Wt_Q(A)$ is the class of witness sets of Q relative to the set A on which Q lives.

Observe that any principal filter lives on the set by which it is generated, and, moreover, this set is its witness set. Atomic quantifiers live on the universe E only and weakly live on their unique elements.

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"Ordinary" determiners denote functions from sets to type $\langle 1 \rangle$ quantifiers. They are thus type $\langle 1, 1 \rangle$ quantifiers.

Accusative extensions of type $\langle 1 \rangle$ quantifiers are specific type $\langle 2 : 1 \rangle$ functions. They satisfy the invariance property called *accusative extension condition* **EC** (Keenan and Westerstahl 1997):

Definition 3. *A type $\langle 2 : 1 \rangle$ function F satisfies **EC** iff for R and S binary relations, and $a, b \in E$, if $aR = bS$ then $a \in F(R)$ iff $b \in F(S)$.*

Observe that if F satisfies **EC** then for all $X \subseteq E$ either $F(E \times X) = \emptyset$ or $F(E \times X) = E$. Given that $SELF(E \times A) = A$ the function $SELF$ does not satisfy **EC**. The function $SELF$ satisfies the following weaker predicate invariance condition **PI** (Keenan 2007):

Definition 4. *A type $\langle 2 : 1 \rangle$ function F is predicate invariant (**PI**) iff for R and S binary relations, and $a \in E$, if $aR = aS$ then $a \in F(R)$ iff $a \in F(S)$.*

This condition is also satisfied for instance by the function $ONLY\text{-}SELF$ defined as follows: $ONLY\text{-}SELF(R) = \{x : xR = \{x\}\}$. Given that $ONLY\text{-}SELF(E \times \{a\}) = \{a\}$, the function $ONLY\text{-}SELF$ does not satisfy **EC**.

The following proposition indicates another way to define **PI**:

Proposition 1. *A type $\langle 2 : 1 \rangle$ function F is predicate invariant iff for any $x \in E$ and any binary relation R, $x \in F(R)$ iff $x \in F(\{x\} \times xR)$.*

The conditions **EC** and **PI** concern type $\langle 2 : 1 \rangle$ functions, considered here as being denoted by "full" verbal arguments or GNPs. Such verbal arguments can be syntactically complex in the sense that they are formed by the application of *generalised determiners* (GDets) to CNs. For instance the GDet *every...except himself* can apply to the CN *student* to give a genuine GNP *every student except himself*. In this case GDets denote type $\langle 1, 2 : 1 \rangle$ functions. Such functions also are constrained by invariance conditions. Thus:

Definition 5. *A type $\langle 1, 2 : 1 \rangle$ function F satisfies **D1EC** iff for R and S binary relations, $X \subseteq E$ and $a, b \in E$, if $aR \cap X = bS \cap X$ then $a \in F(X, R)$ iff $b \in F(X, S)$.*

Observe that if $F(X, R)$ satisfies **D1EC** then for all $X, A \subseteq E$ either $F(X, E \times A) = \emptyset$ or $F(X, E \times A) = E$. Denotations of ordinary determiners occurring in NPs which are in the direct object position satisfy **D1EC**. More precisely, if D is a type $\langle 1, 1 \rangle$ (conservative) quantifier, then the function $F(X, R) = D(X)_{acc}(R)$ satisfies **D1EC**. Indeed, in this case $F(X, R) = \{y : D(X)(yR \cap X) = 1\}$ and $F(X, S) = \{y : D(X)(yS \cap X) = 1\}$. So if $aR \cap X = bS \cap X$ then $a \in F(X, R)$ iff $b \in F(X, S)$.

Functions denoted by properly anaphoric determiners (ones which form GNPs denoting functions satisfying **PI** but failing **EC**) do not satisfy **D1EC**. For instance the function $F(X, R) = \{y : X \cap yR = \{y\}\}$ denoted by the anaphoric determiner *no... except himself/herself* does not satisfy **D1EC**. To see this observe that for $A = \{a\}$ and X such that $a \in X$ one has $F(X, E \times A) = \{a\}$ and thus $F(X, E \times X) \neq \emptyset$ and $F(X, E \times X) \neq E$.

Type $\langle 1, 2 : 1 \rangle$ functions denoted by anaphoric determiners do not satisfy **D1EC**. They satisfy the following weaker condition (Zuber 2010b):

Definition 6. *A type $\langle 1, 2 : 1 \rangle$ function F satisfies **D1PI** (*predicate invariance for unary determiners*) iff for R and S binary relations $X \subseteq E$, and $x \in E$, if $xR \cap X = xS \cap X$ then $x \in F(X, R)$ iff $x \in F(X, S)$.*

The following proposition indicates an equivalent way to define **D1PI**:

Proposition 2. *A type $\langle 1, 2 : 1 \rangle$ function F satisfies **D1PI** iff for any $x \in E$, $X \subseteq E$, any binary relation R one has $x \in F(X, R)$ iff $x \in F(X, (\{x\} \times X) \cap R)$.*

The above invariance principles concern type $\langle 2 : 1 \rangle$ and type $\langle 1, 2 : 1 \rangle$ functions. We need to present similar "higher order" invariance principles for type $\langle 2 : \langle 1 \rangle \rangle$ and type $\langle 1, 2 : \langle 1 \rangle \rangle$ functions that is functions having as output a set of type $\langle 1 \rangle$ quantifiers. This is necessary because, as we will see, some type $\langle 1, 2 : \langle 1 \rangle \rangle$ functions are denotations of RecDets.

One can distinguish various kinds of type $\langle 2 : \langle 1 \rangle \rangle$ and type $\langle 1, 2 : \langle 1 \rangle \rangle$ functions. Observe first that any type $\langle 2 : 1 \rangle$ function whose output is denoted by a VP can be lifted to a type $\langle 2 : \langle 1 \rangle \rangle$ (type $\langle \langle \langle e, t \rangle t \rangle t \rangle$ in Montague notation) function. This is in particular the case with the accusative extensions of a type $\langle 1 \rangle$ quantifier. For instance the accusative extension of a type $\langle 1 \rangle$ quantifier can be lifted to type $\langle 2 : \langle 1 \rangle \rangle$ function in the way indicated in (17). Such functions will be called *accusative lifts*. More generally, if F is a type $\langle 2 : 1 \rangle$ function, its lift F^L, a type $\langle 2 : \langle 1 \rangle \rangle$ function, is defined in (18):

(17) $Q^L_{acc}(R) = \{Z : Z(Q_{acc}(R)) = 1\}$.

(18) $F^L(R) = \{Z : Z(F(R)) = 1\}$.

The variable Z above runs over the set of type $\langle 1 \rangle$ quantifiers.

For type $\langle 2 : \langle 1 \rangle \rangle$ functions which are lifts of type $\langle 2 : 1 \rangle$ functions we have:

Proposition 3. *If a type $\langle 2 : \langle 1 \rangle \rangle$ function F is a lift of a type $\langle 2 : 1 \rangle$ function then for any type $\langle 1 \rangle$ quantifiers Q_1 and Q_2 and any binary relation R, if $Q_1 \in F(R)$ and $Q_2 \in F(R)$ then $(Q_1 \wedge Q_2) \in F(R)$.*

For type $\langle 2 : \langle 1 \rangle \rangle$ functions which are accusative lifts we have:

Proposition 4. *Let F be a type $\langle 2 : \langle 1 \rangle \rangle$ function which is an accusative lift. Then for any $A, B \subseteq E$, any binary relation R, $Ft(A) \in F(R)$ and $Ft(B) \in F(R)$ iff $Ft(A \cup B) \in F(R)$.*

Accusative lifts satisfy the following higher order extension condition **HEC** (Zuber 2014):

Definition 7. *A type $\langle 2 : \langle 1 \rangle\rangle$ function F satisfies **HEC** (higher order extension condition) iff for any natural type $\langle 1 \rangle$ quantifiers Q_1 and Q_2 with the same polarity, any $A, B \subseteq E$, any binary relations R, S, if $Li(Q_1, A)$, $Li(Q_2, B)$ and $\forall_{a\in A}\forall_{b\in B}(aR = bS)$ then $Q_1 \in F(R)$ iff $Q_2 \in F(S)$.*

Functions satisfying **HEC** have the following property:

Proposition 5. *Let F satisfies **HEC** and let $R = E \times C$, for $C \subseteq E$ arbitrary. Then for any $X \subseteq E$ either $Ft(X) \in F(R)$ or for any X, $Ft(X) \notin F(R)$*

Thus a function satisfying **HEC** condition and whose argument is the cross-product relation of the form $E \times A$, has in its output either all principal filters or no principal filter.

It follows from Proposition 5 that lifts of genuine predicate invariant functions do not satisfy **HEC**. They satisfy the following weaker condition (Zuber 2014):

Definition 8. *A type $\langle 2 : \langle 1 \rangle\rangle$ function F satisfies **HPI** (higher order predicate invariance) iff for type $\langle 1 \rangle$ quantifier Q, any $A \subseteq E$, any binary relations R, S, if $Li(Q, A)$ and $\forall_{a\in A}(aR = aS)$ then $Q \in F(R)$ iff $Q \in F(S)$.*

An equivalent way to define **HPI** is given in Proposition 6:

Proposition 6. *Function F satisfies **HPI** iff if $Li(Q, A)$ then $Q \in F(R)$ iff $Q \in F((A \times E) \cap R)$*

The above definitions of **HEC** and of **HPI** easily extend to type $\langle 1, 2 : \langle 1 \rangle\rangle$ functions, which are, as we will see, denotations of RecDets:

Definition 9. *A type $\langle 1, 2 : \langle 1 \rangle\rangle$ function F satisfies **D1HEC** (higher order extension condition for unary determiners) iff for any natural type $\langle 1 \rangle$ quantifiers Q_1 and Q_2 with the same polarity, any $A, B \subseteq E$, any binary relations R, S, if $Li(Q_1, A)$, $Li(Q_2, B)$ and $\forall_{a\in A}\forall_{b\in B}(aR \cap X = bS \cap X)$ then $Q_1 \in F(X, R)$ iff $Q_2 \in F(X, S)$.*

Definition 10. *A type $\langle 1, 2 : \langle 1 \rangle\rangle$ function F satisfies **D1HPI** (higher order predicate invariance for unary determiners) iff for any type $\langle 1 \rangle$ quantifier Q, any $A \subseteq E$, any binary relations R, S, if $Li(Q, A)$ and $\forall_{a\in A}(aR \cap X = aS \cap X)$ then $Q \in F(X, R)$ iff $Q \in F(X, S)$.*

The condition **D1HPI** can also be characterised as in:

Proposition 7. *$F(X, R)$ satisfies **D1HPI** iff if Q lives on A then $Q \in F(X, R)$ iff $Q \in F(X, (A \times X) \cap R)$*

The second series of properties of functions we will discuss concerns conservativity. Recall first the constraint of conservativity for type $\langle 1, 1 \rangle$ quantifiers:

Definition 11. *$F \in CONS$ iff $F(X, Y) = F(X, X \cap Y)$ for any $X, Y \subseteq E$*

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Conservative quantifiers have two important sub-classes: intersective and co-intersective quantifiers (Keenan 1993): a type $\langle 1,1\rangle$ quantifier F is intersective (resp. co-intersective) iff $F(X_1, Y_1) = F(X_2, Y_2)$ whenever $X_1 \cap Y_1 = X_2 \cap Y_2$ (resp. $X_1 \cap Y_1' = X_2 \cap Y_2'$).

All the above properties of quantifiers can be generalised so that they apply to simple and higher order functions (Zuber 2010a):

Definition 12. *A function F of type $\langle 1,2:\tau\rangle$ is conservative iff $F(X,R) = F(X,(E\times X)\cap R)$.*

Definition 13. *A type $\langle 1,2:\tau\rangle$ function is intersective iff $F(X_1,R_1) = F(X_2,R_2)$ whenever $(E\times X_1)\cap R_1 = (E\times X_2)\cap R_2$.*

Definition 14. *A type $\langle 1,2:\tau\rangle$ function is co-intersective iff $F(X_1,R_1) = F(X_2,R_2)$ whenever $(E\times X_1)\cap R_1' = (E\times X_2)\cap R_2'$.*

As in the case of type $\langle 1,1\rangle$ quantifiers it is possible to give other, equivalent, definitions of intersectivity for type $\langle 1,2:\tau\rangle$ functions:

Proposition 8. *F is intersective iff $F(X,R) = F(E,(E\times X)\cap R)$.*

One can notice that intersective and co-intersective functions are conservative. Furthermore, the type $\langle 1,2:1\rangle$ function $F(X,R) = D(X)_{acc}(R)$ and the type $\langle 1,2:\langle 1\rangle\rangle$ function $F(X,R) = D(X)^L_{acc}(R)$ are intersective if D is an intersective type $\langle 1,1\rangle$ quantifier. In Sect. 5 we will additionally define stronger properties of conservativity, intersectivity and co-intersectivity, properties which are displayed by anaphoric but not by ordinary determiners.

Interestingly for functions satisfying **D1PI** or **D1HPI** we have:

Proposition 9. *Any function satisfying **D1PI** or **D1HPI** is conservative.*

Observe that most of the above definitions do not depend on the type τ and thus they apply to type $\langle 1,2:1\rangle$ and type $\langle 1.2:\langle 1\rangle\rangle$ functions.

4 Semantics of Anaphoric Determiners

For simplicity we will consider that reciprocals formed from RecDets give rise only to full (logical) reciprocity. This means, informally, that given a group of participants in an action described by a transitive verb which can be interpreted as involving reciprocity, all members of the group are in this relation with each other. Indeed, it seems that contrary to the interpretation of the full reciprocal *each other* complex reciprocals cannot easily get a weaker interpretation of reciprocity (cf. Dalrymple *et al.* 1998).

As we have seen, we are considering sentences of the form given in (19) - for RefDets and in (20) - for RecDets:

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(19) $NP\ TVP\ RefDet(CN)$

(20) $NP\ TVP\ RecDet(CN)$

In order to present semantics and some formal properties of RefDets and RecDets the first thing we have to do is to determine their grammatical category and the type of functions they denote. This problem is solved for RefDets: since they form reflexive GNPs by applying to a CN and reflexive GNPs denote type $\langle 2:1\rangle$ functions, RefDets denote a type $\langle 1,2:1\rangle$ function. Reciprocal GNPs and RecDets differ in many respects from reflexive GNPs and RefDets respectively. Both these classes also differ from ordinary dets and ordinary NPs. We have already seen some syntactic differences. To see semantic differences between genuine (anaphoric) GNPs and ordinary NPs consider the following examples:

(21) a. Leo and Lea hug each other.
b. Bill and Sue hug each other.

(22) Leo, Lea, Bill and Sue hug each other.

Clearly (21a) in conjunction with (21b) does not entail (22). Thus, given Proposition 3, functions denoted by reciprocal GNPs are not lifts of type $\langle 2:1\rangle$ functions and the conjunction *and* is not understood pointwise. Hence, to avoid the type mismatch and get the right interpretations we will consider that the GNPs *each other* denotes a type $\langle 2:\langle 1\rangle\rangle$ function and RecDets denote type $\langle 1,2:\langle 1\rangle\rangle$ functions.

We can now look at the semantics of anaphoric determiners. We consider first the class of inclusive anaphoric determiners. As we have seen, a frequent form of inclusive RefDets is given in (23), (where *Det* is an ordinary determiners denoting a monotonic (on the second place) type $\langle 1,1\rangle$ quantifier), *CONJ* is a binary operator. The part *CONJ NP* can be omitted. An example of the determiner of the form (23) is given in (6a). Some other examples are given in (24a) and (24b). As these examples show the Boolean operator *CONJ* needs not to be a "simple conjunction":

(23) *Det...including himself CONJ NP*

(24) a. Dan kissed most students including himself, Leo and Lea.
b. Dan hates most monks including himself but not most Japanese (monks).
c. Dan hates ten logicians including himself or Leo.

The functions denoted by RefDets of the from (23) is given in (25), where D is the denotation of *Det*, $\otimes$ - the denotation of *CONJ* and *NP* denotes Q:

(25) $F(X,R)=\{y: y\in X\wedge\langle y,y\rangle\in R\wedge y\in D(X)_{acc}(R)\otimes y\in Q_{acc}(R)\wedge SLi(Q,A)\subseteq X\}$

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To give the semantics of anaphoric RecDets we will use the partition $\Pi(R^S/X)$. Our definitions will be definitions "be cases" which are determined by the fact that the partition $\Pi(R^S/X)$ is atomic, singular or non-trivial. Thus (27) gives the semantics for RecDets of the form (26), where the *Ft(G)NP* is a NP denoting the principal filter generated by the set G and $EXT(X) = \{X\}$:

(26) *Det... including each other CONJ Ft(G)NP*

(27) (i) $F(X,R) = \emptyset$ if $R^S/X = \emptyset$ or $\Pi(R^S/X)$ is atomic
(ii) $F(X,R) = \{Q : Q \in PL \wedge Li(Q,X) \wedge EXT(D(B)) \subseteq Q \otimes Q \in Ft(X \cap G)^L_{acc}(R)\}$ if $\Pi(R^S/X)$ is singular and B is its only block.
(iii) $F(X,R) = \{Q : Q \in PL \wedge Li(Q,X) \wedge \exists_B(B \in \Pi(R^S/X) \wedge Q(D(B)) = 1) \otimes Q \in Ft(X \cap G)^L_{acc}(R)\}$ if $\Pi(R^S/X)$ is non-trivial.

Clause (i) takes into account the fact that NPs like *nobody, no two individuals, no three students*, etc. cannot occur in the subject position of sentences of the form (26). When the partition has only one block B (clause (ii)) then this block is a product relation and only members of the domain of B are in the mutual relation determined by R.

Let us see now functions denoted by exclusive Ref Dets and exclusive RecDets. Various results concerning exclusive RefDets are given in Zuber (2010b). Exclusive determiners denote intersective or co-intersective type $\langle 1,1\rangle$ quantifiers. Such quantifiers form atomic Boolean algebras whose atoms are uniquely determined by sets. More precisely atoms of the intersective algebra are functions At_A such that $At_A(X)(Y) = 1$ iff $X \cap Y = A$ and atoms of the co-intersective algebra are functions At_B such that $At_B(X)(Y) = 1$ iff $X \cap Y' = B$, $(A, B, X, Y \subseteq E)$.

Atoms of intersective and co-intersective algebras are denoted precisely by exclusive dets which have as the complement of *except* a conjonction of proper names. Thus, roughly speaking, exclusive determiners with *No* denote atoms of the intersective algebra and exclusive determiners with *Every* denote atoms of the co-intersective algebra. For instance the determiner *no...except Leo* denotes the atomic intersective quantifier determined by the singleton $\{L\}$ whose only element is Leo and the determiner *every...except Leo and Lea* denotes the atom of co-intersective functions determined by the set composed of Leo and Lea.

Consider now some examples of type $\langle 1,2:1\rangle$ functions and RefDets denoting them (cf. Zuber 2010b). Let At_A be the (intersective or co-intersective) atom determined by the set A. The type $\langle 1.2:1\rangle$ function F_{At_A} given in (28) is an anaphoric function based on the atomic quantifier At_A. Furthermore, if At_A is intersective then F_{At_A} is intersective and if At_A is co-intersective then F_{At_A} is co-intersective:

(28) $F_{At_A}(X,R) = \{x : x \notin A \wedge At_{A\cup\{x\}}(X)(xR) = 1\}$

Let us see some functions which are instances of (28) for illustration. Take the type $\langle 1,1\rangle$ quantifier NO. It is the atomic intersective quantifier determined by the empty set. Thus $A = \emptyset$, $At_\emptyset = NO$ and consequently, given the values of NO, the anaphoric function F_{NO} based on NO is given in (29):

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(29) $F_{NO}(X, R) = \{x : X \cap xR = \{x\}\}$

The function in (29) is the denotation of the RefDet *no...except himself/herself.*

If $At_A = EVERY\text{-}BUT\text{-}\{L\}$ (where $EVERY\text{-}BUT\text{-}\{\mathrm{L}\}(X, Y) = 1$ iff $X \cap Y' = \{L\}$) then the anaphoric function based on $EVERY\text{-}BUT\text{-}\{L\}$ is given in (30). This function is the denotation of the anaphoric determiner *every... except Leo and himself* (if *Leo* refers to L) which occurs in (31):

(30) $F_{EVERY-BUT-\{L\}}(X, R) = \{x : X \cap xR' = \{x, L\}\}$

(31) Dan admires every linguist except Leo and himself.

Thus (28) gives us a class of functions which are denotable by RefDets.

Let us see now the functions denoted by some exclusive RecDets. To do this we will also use the partition $\Pi(R^S/X)$. In (32) we have the function denoted by the reciprocal determiner *no...except each other*:

(32) (i) $F(X, R) = \{Q : Q \in PL \wedge \neg TWO(E) \subseteq Q\}$ if $R^S/X = \emptyset$ or $\Pi(R^S/X)$ is atomic
(ii) $F(X, R) = \{Q : Q \in PL \wedge D(B) \times D'(B) \cap R = \emptyset \wedge B \cap I' = B \wedge EXT((D(B)) \subseteq Q\}$ if $\Pi(R^S/X)$ has B as its only block.
(iii) $F(X, R) = \{Q : Q \in PL \wedge \exists_B(B \in \Pi(R^S/X))\exists_W(W \in Wt_Q(SLi(Q) \wedge (W \times W) \cap I') = B \wedge D(B) \times D'(B) \cap R = \emptyset\}$ if $\Pi(R^S/X)$ is non-trivial.

To illustrate (32) let $R = \{\langle a, b\rangle, \langle b, a\rangle, \langle a, c\rangle, \langle c, d\rangle, \langle d, c\rangle\}$ and $E = X = \{a, b, c, d\}$. In this case $R^S/X = \{B_1, B_2\}$, where $B_1 = \{\langle a, b\rangle, \langle b, a\rangle\}$ and $B_2 = \{\langle c, d\rangle, \langle d, c\rangle\}$ and thus the clause (iii) applies. Consequently $(I_a \wedge I_b) \notin F(X, R)$ - because $\langle a, c\rangle \in R$, and $(I_c \wedge I_d) \in F(X, R)$. If $R = (A \times A) \cap I'$, where $A = X = \{a, b, c\}$ then $\Pi(R^S/X$ is singular with $B = R$ and $D(B) = A$. Hence, given clause (ii) $EXT(A) \in F(X, R)$, $I_a \wedge I_b \wedge I_c) \in F(X, R)$, $I_b \wedge I_c) \in F(X, R)$. In addition, for instance $Q = \neg(I_c \wedge I_d) \in F(X, R)$ because $EXT(A) \subseteq Q$.

To obtain the function denoted by *every... except each other* observe the following equivalence (supposing that *like* is the negation of *dislike*):

(33) Leo and Lea like every student except each other.

(34) Leo and Lea dislike no student except each other.

We can thus consider that the function $G(X, R)$ denoted by *every...except each other* can be obtained from the function $F(X, R)$ denoted by *no... except each other* by changing the relational argument into its Boolean complement: $G(X, R) = F(X, R')$.

5 Formal Properties

The functions described in the previous section are anaphoric in the sense that they satisfy predicate invariance conditions **D1PI** or **D1HPI** and do not satisfy the weaker conditions **D1EC** or **D1HEC**. This is easy to see for functions in (25), (29) and (30). To show that functions denoted by RecDets do not satisfy **D1HEC** we can use Proposition 10, analogous to Proposition 5:

Proposition 10. *Let F satisfies **D1HEC** and let $R = E \times C$, for $C \subseteq E$ arbitrary. Then for any $A \subseteq E$ either $Ft(A) \in F(X, R)$ or for any X, $Ft(A) \notin F(X, R)$*

Using Proposition 10 one can show that function in (32) and the function denoted by *every..., except each* are anaphoric.

Examples of RefDets discussed above suggest that functions they denote satisfy a constraint stronger than conservativity. Observe that the anaphoric functions given in (25), (28) and (29) all have the property given in (35):

(35) $F(X, R) \subseteq X$.

This is also true of denotations of anaphoric determiners formed with *self* and other connectives than *except* or *including*. It is easy to see that the determiner like *five..., in addition to Lea and himself* also denotes a function which satisfies the condition given in (35). We see for instance that in (6a) Lea is a student and in (8) Leo and Lea are linguists.

Interestingly, the anaphoric condition **D1PI** and the condition given in (35) entail a specific version of conservativity, *anaphoric conservativity* (or a-conservativity), specific to non possessive anaphoric determiners. It is defined as follows:

Definition 15. *A type $\langle 1, 2 : \tau \rangle$ function F is* a-conservative *iff $F(X, R) = F(X, (X \times X) \cap R)$.*

The following proposition makes clearer what a-conservativity is:

Proposition 11. *A type $\langle 1, 2 : \tau \rangle$ function F is* a-conservative *iff for any $X \subseteq E$ and any binary relations R_1 and R_2 if $(X \times X) \cap R_1 = (X \times X) \cap R_2$ then $F(X, R_1) = F(X, R_2)$.*

Thus, informally, second, relational arguments of an a-conservative function give rise to different values of the function only if they differ by a specific symmetric part formed from the first argument of the function.

Any a-conservative function is conservative. Ordinary determiners in object position in general do not denote a-conservativce functions: if D is a (conservative) type $\langle 1, 1 \rangle$ quantifier, then the type $\langle 1, 2 : 1 \rangle$ function $F(R, X) = D(X)_{acc}(R)$ is not a-conservative. For instance if $D = ALL$ and $R = E \times A$ then $F(X, R) = ALL(X)_{acc}(E \times A) = E$ if $X \subseteq A$ but in this case $F(X, (X \times X) \cap R) = ALL(X)_{acc}((X \times X) \cap (E \times A) = X$. Thus $F(X, R) \neq F(X, (X \times X) \cap R)$ which means that $F(X, R) = ALL(X)_{acc}(R)$ is not a-conservative (though it is conservative).

Concerning RefDets and a-conservativity we have:

Proposition 12. *A type $\langle 1, 2 : 1 \rangle$ function F satisfying **D1PI** such that $F(X, R) \subseteq X$ is a-conservative.*

Thus the functions denoted by (non-possessive) reflexive anaphoric determiners are a-conservative.

When one looks at type $\langle 1,2 : \langle 1 \rangle \rangle$ functions $F(X,R)$, denotations of non-possessive RecDets, one observes that they have the property given in (36):

(36) If $Q \in F(X,R)$, then Q lives on X.

For instance in (9a) Leo and Lea are vegetarians and thus the quantifier denoted by *Leo and Lea* weakly lives on the set $VEGETARIAN$. Similarly, in (9b) most teachers are Japanese and thus the quantifier $MOST(TEACHER)$ weakly lives on the set $JAPANESE$.

Properties indicated in (35) and (36) are related to the meaning of the connectors *including* and *except.* occurring in non-possessive anaphoric determiners. Possessive anaphoric determiners do not have these properties.

For functions denoted by non-possessive RecDets which satisfy the condition in (36) we have:

Proposition 13. *Any type $\langle 1,2 : \langle 1 \rangle \rangle$ conservative functions satisfying **D1HPI** and the condition in (36) is a-conservative.*

We can thus suppose that *self* and *each other* type anaphoric determiners denote a-conservative functions.

More can be said with respect to the class of functions denoted by anaphoric exclusive determiners. Since they are related either to "ordinary" intersective determiners (like *no... except Leo*) or to "ordinary' co-intersective determiners (like *every... except Lea*) they are provably either intersective or co-intersective (in the sense of definitions D13 and D14 respectively). The function in (32) is intersective and the function denoted by *every..., except each other* is co-intersective.

In addition, given that the functions we consider satisfy predicate invariance and condition like (35) or (36), they have a stronger property than just intersectivity or co-intersectivity: they are a-intersective or a-co-intersective:

Definition 16. *A type $\langle 1,2 : \tau \rangle$ function F is a-intersective iff $F(X_1,R_1) = F(X_2,R_2)$ whenever $(X_1 \times X_1) \cap R_1 = (X_2 \times X_2) \cap R_2$.*

Definition 17. *A type $\langle 1,2 : \tau \rangle$ function F is a-co-intersective iff $F(X_1,R_1) = F(X_2,R_2)$ whenever $(X_1 \times X_1) \cap R'_1 = (X_2 \times X_2) \cap R'_2$.*

The following proposition gives another characterisation of the a-intersectivity and a-co-intersectivity:

Proposition 14. *A type $\langle 1,2 : \tau \rangle$ function F is a-intersective iff $F(X,R) = F(E,(X \times X) \cap R)$.*

Proposition 15. *A type $\langle 1,2 : \tau \rangle$ function F is a-co-intersective iff $F(X,R) = F(E,((X \times X)') \cup R)$.*

Functions which are a-intersective or a-co-intersective are a-conservative. Functions in (29) and in (32) are a-intersective and functions in (30) and the one denoted by *every ..., except each other* are a-cointersective.

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6 Conclusive Remarks

Any discussion of the meaning of (full) reflexives and reciprocals necessitates the use of simple logical tools from the theory of relations. In this paper such tools, in addition to the generalised quantifier theory, have been used to discuss logical properties of anaphoric determiners, that is functional expressions which apply to CNs and form reflexive or reciprocals. Syntactically, anaphoric determiners are discontinuous formatives which contain as their parts "ordinary" determiners and anaphoric pronouns like *himself* or *each other*. This fact entails the proposal made here concerning the logical type of functions denoted by anaphoric determiners: these functions take two arguments: the first argument is a set, because they are denoted by determiners and the second argument is a binary relation because they form simple nominal anaphors. Formal properties of such anaphoric determiners are inherited from the properties of their parts: they are conservative (intersective, co-intersective) because the "ordinary" determiners that compose them are conservative (intersective, co-intersective) and they are predicate invariant because anaphoric pronouns that compose them are predicate invariant. Their anaphoricity is characterised in addition by a-conservativity (a-intersectivity, a-co-intersectivity), a property which is not displayed by "ordinary" determiners.

The results presented in this paper show that though the existence of anaphoric determiners extends the expressive power of NLs because the functions they denote lie outside the class of generalised quantifiers classically defined, these functions resemble quantifiers denoted by "ordinary" nominal determiners in certain important ways.

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