On Generalized Noun Phrases

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Abstract. Generalized noun phrases are expressions which play the role of verbal arguments in the same way as ordinary NPs. However proper generalized NPs cannot easily occur in all argumental positions of the verb. Two types of generalized NPs are distinguished and semantically characterized and various properties of functions they denote are studied. These properties indicate similarities and differences between ordinary NPs and generalized NPs and show that generalized NPs essentially extend the expressive power of natural languages.

1 Introduction

Syntactically, generalized noun phrases (GNPs) belong to expressions which typically fulfill the function of argument of the main clause argument like (ordinary) noun phrases (ONPs). However, genuine GNPs are expressions which, in contrast to ONPs, cannot occur in all argumental positions of the verb; in particular they cannot occur in the subject position. Typical examples of (proper) GNPs are anaphoric NPs (ANPs) whose positions in the sentence are determined by the position of their antecedents. The classical example of an ANP belonging to the sub-class of reflexives is the pronoun *himself* and the classical example of the ANP belonging to the sub-class of reciprocals is the pronoun *each other*.

In section 3 we discuss in some details the structure of GNPs and of ANPs in particular. At present it suffices to indicate that we will count as proper GNPs many complex expressions containing *himself* or *each other*. Such complex examples can in particular be Boolean compounds of anaphoric pronouns with anaphoric or non-anaphoric noun phrases. For instance *himself but not most students* is such a reflexive and *each other and ten philosophers* is such a reciprocal.

There are also complex reflexives and reciprocals which are GNPs which are not Boolean compounds. One can obtain such complex ANPs by the application of anaphoric determiners (ADets) to a common noun (CN) (cf. Zuber 2010b). Thus we have reflexive (anaphoric) determiners (RefDets) like for instance *no,... except herself* or *most,..., including Socrates and himself* which can apply to a CN and give complex reflexives like *no teacher, except herself* or *most philosophers, including Socrates and himself*. Similarly reciprocal determiners (RecDets) like *no... except each other, most..., including each other* which can apply to a CN and give complex reflexives and reciprocals like *every logician except each other* (as in *Dan and Leo admire every logician except each other*).

Another important class of GNPs, distinct from ANPs representing nominal anaphors, is formed by comparative generalised NPs, CNPs. It contains two sub-classes. First, to CNPs belong what Keenan 2016 calls *predicate anaphors*, that is expressions like *more linguists than Dan* or *the greatest number of teachers* (as found in *Leo met more linguists than Dan/the geatest number of teachers*). Clearly such expressions can be used, at least "at the surface" as verbal arguments for some verbal positions.

The second sub-class of CNPs is formed from what we will call, for reasons to be explained below, higher order comparative GNPs (HCNPs). These are expressions like *the same books* or *very different articles and the oldest book in the library.*

We will also discuss HCNPs formed with determiner like *the same number of.* They give rise to the following HCNPs: *the same number of students, almost the same number of articles* etc.

The following examples show that the indicated reflexives, reciprocals and HCNPs are indeed genuine GNPs:

(1) *He(self) admires Dan.

(2) *Each other admires Dan and Leo.

(3) *Most philosophers, including Socrates and himself admire Dan.

(4) *Every logician except each other admire Dan and Leo.

(5) *More students than Dan knows Leo.

(6) *The same articles and the oldest book in the library read Dan and Leo

Even though many HCNPs can occur in the subject position (for instance *the same CN* as in *the same actors played three characters in the movie*) we will consider them, for formal reason to be given below, as genuine GNPs.

The purpose of this paper is to characterize in a preliminary way denotations of GNPs in their opposition to ONPs. We will be mainly interested in formal properties of functions denoted by GNPs. In the next section we recall some basic notions from the generalized quantifier theory and, more importantly, we show how they can be extended so that they apply to denotations of GNPs, more specifically to CNPs, ANPs and HCNPs. In section 3 we indicate various syntactic forms which GNPs can take. In section 4 we give semantics of some basic GNPs and indicate properties of functions representing this semantics.

2 Formal preliminaries

We will consider binary relations and functions over a universe E, assumed to be finite throughout this paper. $D(R)$ denotes the domain of R.The relation I is the identity relation: $I = \{\langle x, y\rangle : x = y\}$. If R is a binary relation and X a set, then $R/X = R \cap (X \times X)$.The binary relation R^S is the greatest symmetric relation included in R, that is $R^S = R \cap R^{-1}$ and $R^{S-} = R^S \cap I'$. If R is an irreflexive symmetric relation (i.e. $R \cap R^{-1} \cap I = \emptyset$) then $\Pi(R)$ is the least fine partition of R such that every one of its blocks is of the form $(A \times A) \cap I'$. A partition is *trivial* iff it contains only one block. Observe that if R is an irreflexive symmetric relation and $\Pi(R)$ is not trivial than every block of $\Pi(R)$ contains at least two elements.

If a function takes only a binary relation as argument, its type is noted $\langle 2 : \tau \rangle$, where τ is the type of the output; if a function takes a set and a binary relation as arguments, its type is noted $\langle 1, 2 : \tau \rangle$. If $\tau = 1$ then the output of the function is a set of individuals and thus its type is $\langle 2 : 1 \rangle$ or $\langle 1, 2 : 1 \rangle$. The function $SELF$, denoted by the reflexive *himself* and defined as $SELF(R) = \{x : \langle x, x \rangle \in R\}$, is of type $\langle 2 : 1 \rangle$ and the function denoted by the anaphoric determiner *every...but himself* is of type $\langle 1, 2 : 1 \rangle$. We will consider here also the case when τ corresponds to a set of type $\langle 1 \rangle$ quantifiers and thus τ equals, in Montagovian notation, $\langle \langle \langle e, t \rangle t \rangle t \rangle$. The type of such functions will be noted either $\langle 2 : \langle 1 \rangle \rangle$ - functions from binary relations to sets of type $\langle 1 \rangle$ quantifiers or $\langle 1, 2 : \langle 1 \rangle \rangle$ - functions from sets and binary relations to sets of type $\langle 1 \rangle$ quantifiers.

Basic type $\langle 1 \rangle$ quantifiers are functions from sets to truth-values. Functions from sets to type $\langle 1 \rangle$ quantifiers are type $\langle 1, 1 \rangle$ quantifiers which are denoted by (nominal) unary determiners. Basic type $\langle 1 \rangle$ quantifiers are denotations of subject NPs. However, NPs can also occur in the direct object position and in this case their denotations do not take sets (denotations of VPs) as arguments but denotations of TVPs (relations) as arguments (Keenan 2016). To account for this eventuality the domain of application of basic type $\langle 1 \rangle$ quantifiers is extended in the way that it contains in addition the set of binary relations. When a quantifier Q acts as a "direct object" we get its *accusative case extension* Q_{acc} (Keenan and Westerståhl 1997):

Definition 1. *For each type* $\langle 1 \rangle$ *quantifier* Q, $Q_{acc}R = \{a : Q(aR) = 1\}$, *where* $aR = \{y : \langle a, y \rangle \in R\}$.

Formally accusative extensions of type $\langle 1 \rangle$ quantifiers are of the same expressive power as type type $\langle 1 \rangle$ quantifiers because the algebra of type type $\langle 1 \rangle$ quantifiers is isomorphic to the algebra of the accusative extensions of type $\langle 1 \rangle$ quantifiers.

Various applications of the notion of the accusative extension of a quantifier are given in Keenan 2016 where in particular it is shown that the accusative extension allows us to avoid recourse to **LF** movement when interpreting NPs in the object position.

A type $\langle 1 \rangle$ quantifier Q is *positive*, $Q \in POS$, iff $\emptyset \notin Q$; Q is *natural* iff either $Q \in POS$ and $E \in Q$ or $Q \notin POS$ and $E \notin Q$; Q is plural, $Q \in PL$, iff if $X \in Q$ then $|X| \geq 2$. Q_A is the atomic quantifier true of just A.

A special class of type $\langle 1 \rangle$ quantifiers is formed by *individuals*: I_a is an individual (generated by $a \in E$) iff $I_a = \{X : a \in X\}$. They are denotations of proper names. More generally, $Ft(A)$, the *(principal) filter generated by the set* A, is defined as $Ft(A) = \{X : X \subseteq E \wedge A \subseteq X\}$. NPs of the form *Every CN* denote principal filters generated by the denotation of *CN*. Meets of two principal filters are principal filters: $Ft(A) \cap Ft(B) = Ft(A \cup B)$.

We will use also the property of *living on* (cf. Barwise and Coper 1981). The basic type $\langle 1 \rangle$ quantifier lives on a set A (where $A \subseteq E$) iff for all $X \subseteq E$, $Q(X) = Q(X \cap A)$. We extend the notion of living on to the type $\langle 2:1 \rangle$ functions. Thus a type $\langle 2:1 \rangle$ function F lives on the relation S iff $F(R) = F(R \cap S)$ for any binary relation R. It is easy to see that Q lives on A iff Q_{acc} lives on $E \times A$.

If E is finite then there is always a smallest set on which a quantifier Q lives. If A is a set on which Q lives we will write $Li(Q, A)$ and the smallest set on which Q lives will be noted $SLi(Q)$. A related notion is the notion of a witness set of the quantifier Q, relative to the set A on which Q lives:

Definition 2. *$W \in Wt_Q(A)$ iff $W \in Q \wedge W \subseteq A \wedge Li(Q, A)$.*

One can see that any principal filter lives on the set by which it is generated, and, moreover, this set is its witness set. Atomic quantifiers live on the universe E only.

Accusative extensions of type $\langle 1 \rangle$ quantifiers are specific type $\langle 2:1 \rangle$ functions. They satisfy the invariance condition called *accusative extension condition* **EC** (Keenan and Westerståhl 1997):

Definition 3. *A type $\langle 2:1 \rangle$ function F satisfies **EC** iff for R and S binary relations, and $a, b \in E$, if $aR = bS$ then $a \in F(R)$ iff $b \in F(S)$.*

Observe that if F satisfies **EC** then for all $X \subseteq E$ either $F(E \times X) = \emptyset$ or $F(E \times X) = E$. Given that $SELF(E \times A) = A$ the function $SELF$ does not satisfy **EC**. The function $SELF$ satisfies the following weaker predicate invariance condition **PI** (Keenan 2007):

Definition 4. *A type $\langle 2:1 \rangle$ function F is predicate invariant (**PI**) iff for R and S binary relations, and $a \in E$, if $aR = aS$ then $a \in F(R)$ iff $a \in F(S)$.*

This condition is also satisfied for instance by the function $ONLY\text{-}SELF$ defined as follows: $ONLY\text{-}SELF(R) = \{x : xR = \{x\}\}$. Given that $ONLY\text{-}SELF(E \times \{a\}) = \{a\}$, the function $ONLY\text{-}SELF$ does not satisfy **EC**.

The following proposition indicates another way to define **PI** (Zuber 2016):

Proposition 1. *A type $\langle 2:1 \rangle$ function F is predicate invariant iff for any $x \in E$ and any binary relation R, $x \in F(R)$ iff $x \in F(\{x\} \times xR)$.*

The **PI** condition is weaker than **EC**. The function $MORE_{S,d}$ which interprets the CNP *more students than Dan* and which is defined as $MORE_{S,d}(R) = \{x : |xR| > |dR|\}$ satisfies another weakening of **EC**, the so-called *argument invariance* condition **AI** (Keenan and Westerståhl 1997) :

Definition 5. *A type $\langle 2:1 \rangle$ function F is argument invariant (**AI**) iff for any binary relation R and $a, b \in E$, if $aR = bR$ then $a \in F(R)$ iff $b \in F(R)$.*

The invariant conditions **EC**, **PI** and **AI** concern type $\langle 2:1 \rangle$ functions, considered here as being denoted by full GNPs. As an illustration we provide a similar definition for type $\langle 1, 2:1 \rangle$ functions denoted by ordinary (nominal) determiners. Thus:

Definition 6. *A type $\langle 1,2:1\rangle$ function F satisfies **D1EC** iff for R and S binary relations, $X \subseteq E$ and $a,b \in E$, if $aR \cap X = bS \cap X$ then $a \in F(X,R)$ iff $b \in F(X,S)$.*

Observe that if $F(X,R)$ satisfies **D1EC** then for all $X, A \subseteq E$ either $F(X, E\times A) = \emptyset$ or $F(X, E \times A) = E$. Denotations of ordinary determiners occurring in NPs which take direct object position satisfy **D1EC**. More precisely, if D is a type $\langle 1,1\rangle$ conservative quantifier, then the function $F(X,R) = D(X)_{acc}(R)$ satisfies **D1EC**: in this case $F(X,R) = \{y : D(X)(yR\cap X) = 1\}$ and $F(X,S) = \{y : D(X)(yS\cap X) = 1\}$. So if $aR\cap X = bS\cap X$ then $a \in F(X,R)$ iff $b \in F(X,S)$.

The above invariance principles concern type $\langle 2:1\rangle$ and type $\langle 1,2:1\rangle$ functions. We need to present similar "higher order" invariance principles for type $\langle 2:\langle 1\rangle\rangle$ functions, that is functions having as output a set of type $\langle 1\rangle$ quantifiers.

One can distinguish various kinds of type $\langle 2:\langle 1\rangle\rangle$ functions. Observe first that any type $\langle 2:1\rangle$ function whose output is denoted by a VP can be lifted to a type $\langle 2:\langle 1\rangle\rangle$ function. The accusative extension of a type $\langle 1\rangle$ quantifier Q can be lifted to type $\langle 2:\langle 1\rangle\rangle$ function in the way indicated in (7). Such functions will be called *accusative lifts*. More generally, if F is a type $\langle 2:1\rangle$ function, its lift F^L, a type $\langle 2:\langle 1\rangle\rangle$ function, is defined in (8):

(7) $Q^L_{acc}(R) = \{Z : Z(Q_{acc}(R)) = 1\}$.

(8) $F^L(R) = \{Z : Z(F(R)) = 1\}$.

The variable Z above runs over the set of type $\langle 1\rangle$ quantifiers.

For type $\langle 2:\langle 1\rangle\rangle$ functions which are lifts of type $\langle 2:1\rangle$ functions we have:

Proposition 2. *If a type $\langle 2:\langle 1\rangle\rangle$ function F is a lift of a type $\langle 2:1\rangle$ function then for any type $\langle 1\rangle$ quantifiers Q_1 and Q_2 and any binary relation R, if $Q_1 \in F(R)$ and $Q_2 \in F(R)$ then $(Q_1 \wedge Q_2) \in F(R)$*

For type $\langle 2:\langle 1\rangle\rangle$ functions which are accusative lifts we have:

Proposition 3. *Let F be a type $\langle 2:\langle 1\rangle\rangle$ function which is an accusative lift. Then for any $A,B \subseteq E$, any binary relation R, $Ft(A) \in F(R)$ and $Ft(B) \in F(R)$ iff $Ft(A\cup B) \in F(R)$.*

Accusative lifts satisfy the following higher order extension condition **HEC** (Zuber 2014):

Definition 7. *A type $\langle 2:\langle 1\rangle\rangle$ function F satisfies **HEC** (higher order extension condition) iff for any natural type $\langle 1\rangle$ quantifiers Q_1 and Q_2 with the same polarity, any $A,B \subseteq E$, any binary relations R,S, if $Li(Q_1,A)$, $Li(Q_2,B)$ and $\forall_{a\in A}\forall_{b\in B}(aR = bS)$ then $Q_1 \in F(R)$ iff $Q_2 \in F(S)$.*

Functions satisfying **HEC** have the following property::

Proposition 4. *Let F satisfies **HEC** and let $R = E\times C$, for $C \subseteq E$ arbitrary. Then for any $X \subseteq E$ either $Ft(X) \in F(R)$ or for any X, $Ft(X) \notin F(R)$*

Thus a function satisfying **HEC** condition and whose argument is the cross-product relation of the form $E \times A$, has in its output either all principal filters or no principal filter. We will see that the function denoted by the ANP *each other* does not satisfy **HEC**.

It follows from Proposition 4 that lifts of genuine predicate invariant functions do not satisfy **HEC**. They satisfy the following weaker condition (Zuber 2014):

Definition 8. *A type $\langle 2 : \langle 1 \rangle\rangle$ function F satisfies **HPI** (higher order predicate invariance) iff for type $\langle 1 \rangle$ quantifier Q, any $A \subseteq E$, any binary relations R, S, if $Li(Q, A)$ and $\forall_{a \in A}(aR = aS)$ then $Q \in F(R)$ iff $Q \in F(S)$.*

The higher order property corresponding to **AI** is the higher order argument invariance:

Definition 9. *A type $\langle 2 : \langle 1 \rangle\rangle$ function F satisfies **HAI** (higher order argument invariance) iff for any natural type $\langle 1 \rangle$ quantifiers Q_1 and Q_2 with the same polarity, any $A, B \subseteq E$, any binary relation R, if $SLi(Q_1, A)$, $SLi(Q_2, B)$ and $\forall_{a \in A}\forall_{b \in B}(aR = aS)$ then $Q_1 \in F(R)$ iff $Q_2 \in F(R)$.*

Higher order invariance conditions are generalizations of "simple" invariance conditions because it can be shown (cf. Zuber 2014) that lifts of functions satisfying simple invariance condition satisfy higher order invariance conditions. Thus the accusative lift of a type $\langle 1 \rangle$ quantifier satisfies **HEC**, the lift a a function satisfying **PI** satisfies **HPI** and the lift of a function satisfying **AI** satisfies **HAI**.

3 Structure of generalized noun phrases

In this section we indicate some structural and syntactic differences and similarities between ONPs and GNPs by comparing their respective structures. The remarks which follow are not intended, however, to characterise syntactically the class of proper GNPs. Moreover, we have to keep in mind that we consider that the class of GNPs is strictly included in the class of NPs and thus that there are genuine, or proper, GNPs which are not ONPs. In the next section we will characterise semantically two classes of proper GNPs: simple and higher order GNPs. Roughly speaking, simple GNPs are GNPs related to reflexives or simple comparatives (or predicate anaphors) and higher order GNPs are those which are related to reciprocals or HCGNPs such as *the same CN*.

The first thing to notice is that among genuine GNPs there are no elements corresponding to proper names, which, obviously are OMPs. Thus there are no morphologically simple non-pronominal genuine GNPs. We observe that morphologically simple or "almost simple" genuine GNPs have a pronominal character. This is the case with the reflexive *himself* or reciprocal *each other*. Interestingly "ordinary" pronouns are ONPs.

One of very productive ways of forming complex ONPs is by the application of determiners to CNs. Thus there is a natural class of ONPs which are of the form *Det CN* where *Det* is an unary determiner that is a functional expression which when applied to one CN gives an NP. Such "ordinary" determiners have been extensively studied, various sub-classes of them have been distinguished and formal properties of their denotations, that is type $\langle 1, 1 \rangle$ quantifiers, have been established. It is generally admitted that unary determiners denote conservative type $\langle 1, 1 \rangle$ functions and conservative functions have formally important sub-classes of intersective and co-intersective functions. For instance the determiner *most* denotes a conservative function which is conservative but neither intersective nor co-intersective, the numerals are determiners which denote intersective functions and determiners like *every* or *every...but ten* denote co-intersective functions.

Now, important point is that there is also a class of genuine GNPs which are obtained by the application of a (generalised) determiner to a CN, that is GNPs of the form *GDet CN*, where *GDet*, a generalized determiner, is a functional expression which when applied to a CN gives a genuine GNP. GDets in their turn can be divided into anaphoric GDets (ADets) and comparative GDets (CGDets). Finally, among ADets we have RefDets, reflexive determiners and RecDets, that is reciprocal determiners. To see these differrent classes of GDets consider the following examples:

(9) Dan hates every linguist except himself

(10) Dan knows more linguists than Leo

(11) Leo and Dan admire no linguist except each other

(12) Leo and Dan read the same books

In (9) the determiner *every... except himself* is a RefDet. Similarly *no...except himself and Dan* and *most..., including himself* are RefDets. In (10) we have a CDet *more... than Leo.* In (11) the expression *no... except each other* is a RecDet as are for instance expressions like *every... except each other and Dan* or *most, including each other.* In (12) we have a CDet *the same.* Other examples of such determiners are represented by *different, very different, quite different, similar, very similar, almost the same.* etc.

Another very productive way of forming NP is by the use of Boolean connectors. For instance the following NPs are such Boolean compounds *Dan and most students, ten logicians and some linguists, five students and no teacher except Dan.* As the following examples show there are also genuine GNPs which are Boolean compounds:

(13) Dan admires himself and most philosophers

(14) Leo and Dan admire each other but not themselves

(15) Leo and Dan read five articles and the same books

(16) Leo and Dan read the same articles and different books

(17) Leo and Dan admire each other but not themselves and Lea

(18) Dan and Leo admire each other, themselves and the same linguists

Observe that in the above examples GNPs belonging to various subclasses are conjoined. This does not mean that all GNPs can be freely conjoined in the same way as ONPs cannot always be freely conjoined.

The next similarity between ONPs and genuine GNPs I mention briefly concerns the possibility of their modification by the so-called categorially polyvalent modifiers, CPM, that is modifiers which can apply to expressions of different categories. CPM include expressions like *only, also, even, at least*, etc. These modifiers can modify expressions of various categories and in particular they can apply to ONPs since we have: *even Dan, only Dan and Bill, also some students, at least ten teachers*, etc. As the following examples show, proper GNPs can also be modified by the CPM:

(19) Leo admires only/even/at/ least himself.

(20) Leo and Dan admire at least/at most/only/even each other.

(21) They read even/at least the same books.

Finally, ONPs and proper GNPs can also play the role of arguments of non-verbal predicatives, that is complex expressions which are not modifiers and do not contain a verb but which take GNPs as an argument. A natural class of such predicatives is formed eiher from transitive CNs (like *friend of* or *young grand-parent of*) or from transitive adjectives (like *jealous of*). Thus we have the following predicatives in which ONPs occur as arguments: *grand-parents of ten children, friends of some gangsters*. Proper GNPs can also occur in such contexts since we have: *grand-parents of the same students*, *fond of the same students/themselves* or *jealous of each other*.

Proper GNPs can also occur in relative clauses and other embedding constructions. In this case, however, the complex constructions containing such embedded GNPs can easily occur in the subject position: the expressions *persons with the same taste/who admire each other* and *to hate each other* can be used as subject NPs. In addition both types of noun phrases can occur as arguments in prepositional phrases since we have *leave with each other* or *talk with the same persons*.

A general form of sentences in which ANPs, CNPs and HCNPs occur and which we will consider here, is given in (22):

(22) $NP\ TVP\ GNP$

TVP is a transitive verb phrase which denotes a binary relation and GNP is either *himself* or *each other* or has the form RefDet(CN), RecDet(CN), *the same CN* or is a Boolean combination of all such cases.

4 Formal properties of generalized noun phrases

In this section we analyze properties of full GNPs and not of their specific parts such as reflexive determiners, comparative determiners or reciprocal determiners. Properties of RefDets are studied in Zuber 2010b and comparative determiners

- in Zuber 2011. A proposal to treat some higher order comparatives is given in Zuber 2017b.

The fact that nominal anaphors have various properties which distinguish them from ONPs is well known. For instance Geach 1968 indicates various peculiarities of the pronoun *himself* and suggest that some of them have been discussed by medieval philosophers. I will illustrate first informally some differences between ONPs and various simple proper GNPs, using in particular various observations from Keenan 2007, Keenan 2016 and Zuber 2011.

When an ONP (in the subject position) denotes a type $\langle 1 \rangle$ quantifier Q then when it occurs in the object position it denotes the accusative extension Q_{acc} of Q. Accusative extensions of a quantifier satisfy **EC**. This means that if, for instance, the persons that Leo washes are the same as the persons that Dan shaves than the following two sentence forms have the same truth values, for any NP:

(23) Leo washes NP

(24) Dan shaves NP

This is not the case with functions denoted by simple proper GNPs. Consider for instance the reflexive *himself*. Suppose again that persons that Dan washes are the same as the persons that Leo shaves. In this case the following sentences can fail to have the same truth value:

(25) Dan washes himself.

(26) Leo shaves himself.

Consider now the CNP *the greatest number of languages*. Suppose that the set of languages that Dan speaks is the same as the set of languages that Leo studies. It does not follow from this that the following sentences have the same truth value:

(27) Dan speaks the greatest number of languages.

(28) Leo studies the greatest number of languages.

Thus simple GNPs denote functions which do not satisfy **EC** satisfied by accusative extensions denoted by ONPs in the object position. Functions denoted by reflexives satisfy the weaker **PI** condition and functions denoted by simple comparatives satisfy the weaker **AI** condition.

Higher order GNPs are additionally different from ONPs and from simple GNPs. To see this informally consider the following examples (cf. Zuber 2014):

(29) a. Leo and Lea hug each other/read the same books.
b. Bill and Sue hug each other/read the same books.

(30) Leo, Lea, Bill and Sue hug each other/read the same books.

Clearly (29a) in conjunction with (29b) does not entail (30). However, if we replace *each other* or *the same books* by an ordinary NP or by a simple proper GNP, the corresponding entailment holds. This means that the conjunction *and* cannot be understood pointwise and that the functions denoted by GNPs like *the same CN* and *each other* are of not the same type as functions denoted by ONPs or by simple GNPs.

Observe that the non entailment of (30) from (29a) and (29b) in conjunction with Proposition 3 indicates that the GNPs *the same books* and *each other* are not accusative lifts (of any type $\langle 1 \rangle$ quantifier).

The question one can ask now is what is the logical type of the result of the function denoted by GNPs which are reciprocals or HCNPs. We know that sentences with such GNPs (in the object position) do not take proper nouns as subjects and thus the type of objects denoted by the subject NP cannot be e, the type corresponding to individuals. We can suppose that it is of the raised type $\langle\langle e,t\rangle,t\rangle$, which, ignoring directionality, corresponds to the category $S/(S/NP)$. Since *the same CN* and *each other* form a verbal argument playing the role of direct object, *the same (CN)* and *each other* apply to a transitive verb to form a VP. Semantically, this verb phrase denotes a set of type $\langle 1 \rangle$ quantifiers. Thus, in order to avoid the type mismatch, the verb phrase must be raised to become of the category $S/(S/(S/NP))$. This can be done using the following higher order reduction *via* function application (where "+" symbolises the function application):

(31) $S/(S/NP)+\ S(S/(S/NP)) = S$

Thus in (31) the VP has been raised to the category $S(S/(S/NP))$ whose type is now $\langle\langle\langle e,t\rangle t\rangle t\rangle$. This means that such raised VPs denote a set of type $\langle 1 \rangle$ quantifiers and consequently the sentence of the form (22) is true iff the quantifier denoted by the NP belongs to the set denoted by the $TVP\ GNP$. Keenan and Faltz 1985 show that (extensional) non-raised VPs (classically) denote, up to the isomorphism, specific characteristic functions of sets of type $\langle 1 \rangle$ quantifiers, that is they denote a set of quantifiers. The reason is that these characteristic functions are in addition homomorphisms (from the algebra of quantifiers to the algebra of truth-values) and the algebra of such homomorphic functions is isomorphic to the algebra of sets (subsets of the universe), the classical denotational domain of VPs (or of one-place predicates in the first order logic). Thus "classical" denotations of VPs are homomorphic in the sense that they preserve meets in particular. We have seen that this is not the case for the VPs formed from higher order GNPs given the non-entailment between (29) and (30) and consequently VPs with higher order GNPs do not denote sets, subsets of E, but sets of type $\langle 1 \rangle$ quantifiers.

Let me start the discussion of formal properties by the following:

Proposition 5. *Boolean algebra of type $\langle 1 \rangle$ quantifiers is isomorphic to the algebra of intersective type $\langle 1,1 \rangle$ quantifiers and to the algebra of co-intersective quantifiers.*

The proof of this proposition is obvious if one observes that there is one to one correspondence between atoms of the algebra of type $\langle 1 \rangle$ quantifiers and the atoms of the algebra of intersective quantifiers. Indeed for any set A the singleton $\{A\}$ is an atomic type $\langle 1 \rangle$ quantifier and the type $\langle 1, 1 \rangle$ quantifier F_A such that $F_A(X)(Y) = 1$ iff $X \cap Y = A$ is an atom of the algebra of intersective quantifiers.

Thus there are as many type $\langle 1 \rangle$ quantifiers as there are intersective quantifiers. Since (cf. Keenan and Westerståhl 1997) any type $\langle 1 \rangle$ quantifier is expressible (in English) by an ONP (of English) this means that there as many (English) ONPs as there are intersective (or co-intersective) quantifiers. If the universe E has n elements then there are 2^k, for $k = 2^n$ type $\langle 1 \rangle$ quantifiers. But there are much more anaphoric type $\langle 2 : 1 \rangle$ functions. As Keenan 2007 indicates in this case there are 2^m, for $m = n \times 2^n$ functions satisfying **PI**. The number off all functions from the set of binary relations to the set of sets equals k^l for $k = 2^n$ and $l = 2^{n \times n}$. This means that in the universe with just two elements there are 16 type $\langle 1 \rangle$ quantifiers, 256 functions satisfying **PI** and 4^{16} functions from binary relations to sets.

Let us now define some functions denoted by some GNPs. To define the type $\langle 2 : \langle 1 \rangle \rangle$ function EA denoted by the reciprocal *each other* we use the partition $\Pi(R^{S-})$ (Zuber 2016). Our definition is the definition "by cases" which depend on whether the partition $\Pi(R^{S-})$ is trivial or non-trivial. Thus

Definition 10.
(i) $EA(R) = \{Q : Q \in PL \wedge \neg 2(E) \subseteq Q\}$ *if* $R^{S-} = \emptyset$
(ii) $EA(R) = \{Q : Q \in PL \wedge Q_{D(B)} \subseteq Q\}$*, if* $\Pi(R^{S-})$ *is trivial with* B *as its only block*
(iii) $EA(R) = \{Q : Q \in PL \wedge \exists_B(B \in \Pi(R^{S-}) \wedge Q(D(B) = 1\} \cup \{Q : Q \in PL \wedge \exists_B(B \in \Pi(R^{S-}) \wedge Q = \neg Q_{D(B)}\}$ *if* $\Pi(R^{S-})$ *is non-trivial.*

Functions denoted by GNPs are anaphoric in the sense that they satisfy predicate invariance conditions **PI** or **HPI** and do not satisfy stronger conditions **EC** or **HEC**. We have already seen that $SELF$ and $ONLY\text{-}SELF$ are anaphoric in that sense. Using proposition 5 and definition 8 we show that the function EA in definition (10) is anaphoric (because for $R = E \times A$ the partition $\Pi(R^{S-})$ is trivial). Some other higher order anaphoric functions are discussed in Zuber 2016 and Zuber 2017a. In particular RefDets and RecDets and properties of functions they denote are discussed in Zuber 2017a.

To define the functions $SAME(X, R)$ and $SAME\text{-}N$ denoted by *the same CN* and *the same number of CN* respectively, where CN denotes X, we will use the set partitions defined by the following equivalence relations (Zuber 2017b):

Definition 11. *(i)* $e_R = \{\langle x, y \rangle : xR = yR\}$
(ii) $e_{R,n} = \{\langle x, y \rangle : card(xR) = card(yR)\}$

We will say that the block of a partition is singular if it is a singleton. A block B is plural, $B \in PL$, if it is contains at least two elements. A partition is atomic iff all its blocks are singular. With the help of these notions, using the partition

$\Pi_{R_A}(E)$ we can now express the function $SAME(X,R)$, where R is a binary relation, as follows (for X and R non-empty and where R_X is a subrelation of R whose range is restricted to X):

Definition 12. $SAME(X,R) =$
(i)$=\{Q : Q \in PLR \wedge \neg 2(E) \subseteq Q\}$, *if* $\Pi_{R_X}(E)$ *is atomic*
(ii)$= \{Q : Q \in PLR \wedge \exists_B(B \in PL \wedge B \in \Pi_{R_X}(E) \wedge Q(B) = 1\} \cup$
$\cup\{Q : Q \in PLR \wedge \exists_{C \subseteq E} \forall_{B \in \Pi_{R_X}(E)} C \not\subseteq B) \wedge \neg ALL(C) \subseteq Q)\}$, *if* $\Pi_{R_X}(E)$ *is not atomic.*

The above definition says that $SAME$ applied to a set X and a binary relation R gives as result a set of quantifiers. This set can be decomposed into various sub-sets depending on the structure of the partition of E induced by R and X. Clause (i) says that when the partition is atomic then no two objects are in the relation R with all objects of a sub-set of X. This entails that the quantifier denoted by *no two objects* and any of its consequences belong to the set $SAME(X,R)$. This means that, for instance, the quantifiers denoted by *no five objects* or *no two students* also belong to the set $SAME(X,R)$.

Clause (ii) concerns the case where the partition is not atomic. In this case there is at least one plural block of the partition such that all its members are, roughly speaking, in the relation R with the same subset of X. This block corresponds to the property expressing the sameness we are looking for and a plural quantifier can be true or false of it. The second part of the clause (ii) provides a set of quantifiers obtained from a "negative information" given by sets which are not blocks of the partition. If, for instance, Jiro and Taro are Japanese students who read different books then no set to which they belong is a block of $\Pi_{R_B}(E)$ - where R corresponds to $READ$ and B - to $BOOK$. Then, according to the second part of the clause (ii), the quantifiers denoted by the *NPs not all Japanese students*, *not all students* and *not all Japanese* belong to $SAME(B,R)$.

The definition of the function $SAME$-N denoted by the generalized determiner *the same number of* is quite similar to the definition of the function $SAME(X,R)$. We just have to replace everywhere in definition 12 the partition $\Pi_{R_X}(E)$ by the partition $\Pi_{R_X,n}(E)$. Consequently we have:

Definition 13. $SAME$-$N(X,R)$=
(i)$=\{Q : Q \in PLR \wedge \neg 2(E) \subseteq Q\}$, *if* $\Pi_{R_{X,n}}(E)$ *is atomic*
(ii)$= \{Q : Q \in PLR \wedge \exists_B(B \in PL \wedge B \in \Pi_{R_{X,n}}(E) \wedge Q(B) = 1\} \cup$
$\cup\{Q : Q \in PLR \wedge \exists_{C \subseteq E}(C \notin \Pi_{R_X}(E) \wedge \neg ALL(C) \subseteq Q)\}$, *if* $\Pi_{R_X}(E)$ *is not atomic.*

Definitions 12 and 13 provide the readings of *the same* and *the same number of* without the existential import that is without the presupposition that the set denoted by *CN* is not empty. In order to get the reading in which the existential import is involved the following equivalence relations have to be used:

Definition 14. $e_R^{ei} = \{\langle x, y\rangle : (xR = yR \wedge xR \neq \emptyset) \vee (x = y)\}$

Definition 15. $e^{ei}_{R,n} = \{\langle x, y\rangle : (|xR| = |yR| \wedge xR \neq \emptyset) \vee (x = y)\}$

The relation e^{ei}_R defines the partition $\Pi^{ei}_R(E)$ and the relation $e^{ei}_{R,n}$ defines the partition $\Pi^{ei}_{R,n}(E)$. It follows from definitions 14 and 15 that if $aR = \emptyset$, then the singleton $\{a\}$ is a singular block of both partitions Π^{ei}_R and $\Pi^{ei}_{R,n}$ and thus is not a member of any plural quantifier. Consequently the reading of *the same* with the existential import is given in definition 16:

Definition 16. $SAME^{ei}(X, R)=$
(i)$=\{Q : Q \in PLR \wedge \neg 2(E) \subseteq Q\}$, *if* $\Pi^{ei}_{R_X}(E)$ *is atomic*
(ii)$= \{Q : Q \in PLR \wedge \exists_B(B \in PL \wedge B \in \Pi^{ei}_{R_X}(E) \wedge Q(B) = 1\}\cup$
$\cup\{Q : Q \in PLR \wedge \exists_{C\subseteq E}\forall_{B\in\Pi^{ei}_{R_X}(E)} C \not\subseteq B) \wedge \neg ALL(C) \subseteq Q)\}$, *if* $\Pi^{ei}_{R_X}(E)$ *is not atomic.*

It is easy, though tedious, to show that functions defined in definitions 12, 13 and 16 satisfy **HAI** (and do not satisfy **HEC**). Consequently, higher order GNPs also denote functions which are not accusative extensions of type $\langle 1\rangle$ quantifiers.

5 Conclusive remarks

Generalized noun phrases are expressions which, syntactically play the role of direct objects as do ordinary NPs. Semantically, however, they do not denote type $\langle 1\rangle$ quantifiers or their accusative extensions. Functions they denote satisfy weaker conditions than the extension condition, which is satisfied by accusative extensions of type $\langle 1\rangle$ quantifiers. In spite of that they resemble quantifiers in various ways.

We distinguished two types of GNPs, according to the type of functions they denote: simple GNPs (for instance reflexives and predicate anaphors) denote type $\langle 2 : 1\rangle$ functions and higher order GNPs (like reciprocals) denote type $\langle 2 : \langle 1\rangle\rangle$ functions. Both types of these functions satisfy similar invariance conditions and both types of GNPs have their syntactic structure similar to the structure of ONPs.

Syntactic similarity in the structures of GNPs and ONPs and the fact that the two types of expressions, ONPs and GNPs can occur as different conjuncts in the same Boolean compounds indicates that GNPs should not be considered as a new syntactic category. Rather, to account for the specificity of their semantics we should consider, in the spirit of Partee 1986 that the type of NPs can change depending on the environment it finds itself in. In this case higher order GNPs give rise to the VP raising.

Formal properties of GNPs presented in this paper shows that the existence of anaphors and higher order comparative NPs strongly extends the expressive power of NLs. Keenan 2007 and Keenan 2016 shows that denotations of reflexive anaphors and predicate anaphors lie outside the class of classically defined generalized quantifiers (they do not satisfy the extension condition). Results presented in this paper show that in addition higher order GNPs form non-homomorphic predicates which force the VP raising because because their denotations are not lifts of type $\langle 2 : 1\rangle$ functions.

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