

# A computational approach to the abstraction of morphophonological alternations

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Typologie et modélisation des systèmes morphologiques  
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# A computational abstractive approach

- ▶ Item and Pattern Morphology : Morphology is modeled directly in terms of surface alternations between wordforms.
  - ▶ Term due to **Blevins2016**; preferable to the ambiguous ‘Word and Paradigm’
  - ▶ Abstractive in Blevins, 2006’s sense.
- ▶ Items are wordforms rather than subword units.
- ▶ The **alternation patterns** allow us to classify lexemes’ behavior.
- ▶ Extracting patterns automatically without prior knowledge on the language’s profile is a fundamental step for instrumented typology

# Finding alternation patterns

- ▶ We describe an automated way to find morphophonological alternations cross-linguistically from surface forms.
- ▶ These take the form of bidirectional alternation patterns :  
*X alternates with Y in the context Z* written "X  $\rightleftharpoons$  Y / Z"
- ▶ They can be used to study various alternations

Inflection: English past

'acted'  $\rightleftharpoons$  'act'  $\rightarrow$  id  $\rightleftharpoons$  / X+[bdpt]\_

/aktɪd/  $\rightleftharpoons$  /akt/

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Derivation: French nouns

'gloire'  $\rightleftharpoons$  'glorieux'

/glwaʁ/  $\rightleftharpoons$  /glɔʁjø/  $\Rightarrow$  wa\_  $\rightleftharpoons$  o\_jø / X+\_ɹ\_

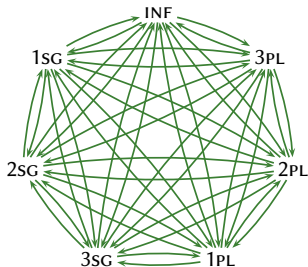
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Variation and Change: French and Morisien

'judiciaire'  $\rightleftharpoons$  'zidisier'  
 /ʒydisjɛʁ/  $\rightleftharpoons$  /zidisjɛʁ/  $\rightarrow$   $\exists y\_ɛ \rightleftharpoons z i\_ɛ / X^* \_ X+ \_$

- ▶ This work was developed with inflectional morphology in mind.
  - ▶ Investigations on Inflection Classes (my PhD topic)
  - ▶ Studying the Paradigm Cell Filling Problem
- ▶ For this purpose, we were considering all possible pairwise alternations in large inflectional paradigms
  - ▶ Here: 7 cells, 42 relations.
- ▶ We needed a system able to find patterns in a reasonable time for a large number of alternations.



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# Abstracting patterns from pairs of forms

- ▶ Input :
  - ▶ Pairs of forms involved in some morphophonological alternation  
ex: /amɛn/  $\rightleftharpoons$  /amøne/
  - ▶ A definition of phonemes in distinctive features  
ex: m = [+son -syl +cons +cont +nas -haut -arr +ant -cor +vois ]



# Abstracting patterns from pairs of forms

- ▶ Input :
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ex: /**amɛn**/ ⇔ /**amøne**/
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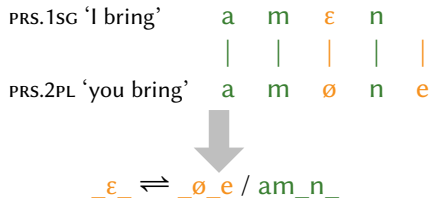
# Patterns can be deduced from alignments

A pattern is a representation of alternating and constant material in a pair of forms.

PRS.1SG 'I bring'	a	m	ε	n	
PRS.2PL 'you bring'	a	m	∅	n	e

# Patterns can be deduced from alignments

A pattern is a representation of alternating and constant material in a pair of forms. It can be deduced from an alignment of the forms



# Finding the optimal local alignment

- Suffixal alternations require left alignment (French):

PRS.1SG 'I bring'	a	m	ε	n	
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- ▶ Prefixal alternations require a right alignment (Swahili):

FUT.1SG 'I will want'	n	i	t	a	t	a	k	a
PRS.2PL 'you want'		m	n	a	t	a	k	a

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- ▶ More complex alternations can require arbitrary alignments :

PRS.1SG 'I buy'	m	i	x	a	r		a	n	(Persian)

PAS.PERF.1SG 'I bought'			x	a	r	i	d	a	n
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PFV.3SG 'he wrote'		k	a:	t	a	b	a:	(Arabic)

IPF.3SG 'he writes'	j	u	k	a:	t	i	b	u
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# Finding the optimal global alignment

- ▶ There can be several optimal local alignments

Alignments for 'baba' and 'ba'

	Alignment				Pattern
	b	a	b	a	
(i) Prefix	_	_	b	a	$\epsilon \rightleftharpoons$ ba / _ba
(ii) Suffix	b	a	_	_	$\epsilon \rightleftharpoons$ ba / ba_
(iii) Infix	b	_	_	a	$\epsilon \rightleftharpoons$ ab / b_a

# Finding the optimal global alignment

- ▶ There can be several optimal local alignments
- ▶ Choosing the most appropriate alignment requires cross-lexeme comparisons

Three imaginary languages.

A. Infix -ab-		B. Prefix ba-		C. Suffix -ba	
SG	PL	SG	PL	SG	PL
ba	baba	ba	baba	ba	baba
to	tabo	to	bato	to	toba
ri	rabi	ri	bari	ri	riba
su	sabu	su	basu	su	suba
ne	nabe	ne	bane	ne	neba

# Morphology beyond affixes

- ▶ We saw that non concatenative processes can require complex alignments,
- ▶ So do multidimensional alternations, e.g. in Zenzontepec Chatino
  - ▶ In the following example, inflection relies on both segmental and tonal alternations.
  - ▶ It is important to be able to capture both with the same system.

	‘break’	‘drench’	‘slide’	
POT	ku <sup>0</sup> ki <sup>0</sup> tɛ <sup>1</sup> ʔ	ku <sup>0</sup> ki <sup>0</sup> li <sup>0</sup>	ki <sup>0</sup> ki <sup>0</sup> li <sup>0</sup>	...
CPL	nka <sup>0</sup> ki <sup>1</sup> tɛ <sup>2</sup> ʔ	nka <sup>0</sup> ki <sup>0</sup> li <sup>0</sup>	nku <sup>0</sup> ki <sup>0</sup> ti <sup>0</sup>	...

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# The PCFP as a NLP problem

- ▶ When constructing lexical resources, the paradigms extracted from corpora are usually sparse
- ▶ The Paradigm Cell Filling Problem arises in this context  
Finding alternation rules provides a way to fill-in the missing forms.
- ▶ In NLP, this is called reinflection Cotterell et al., 2016.
- ▶ Symbolic solutions have various contributions to the alignment problem.
- ▶ Neural networks perform best (Kann and Schütze, 2016).
- ▶ Because of distinct goals, none provide the type of generalisations we need.

# Studies of inflectional morphology

- ▶ Much previous studies of Inflectional structure (e.g. Ackerman and Malouf 2013; Stump and Finkel 2013) works from hand-designed classifications.
- ▶ Neural networks have been used too as a model of the PCFP (Malouf, 2016).
- ▶ The main contribution to automated inference of patterns is the Minimal Generalization Learner
  - ▶ Simulate the behavior of speakers in a wug test (Albright and Hayes, 2002, 2003, 2006)
  - ▶ Generalize patterns incrementally and retain all intermediate generalizations.
  - ▶ This is both too slow to be applied to large inflection systems and contrary to our need to find one pattern per pair of forms.
- ▶ Following Albright & Hayes, several studies infer alternation patterns with strategies devised for specific datasets (Sims, 2015; Albright and Hayes, 2002; Bonami and Boyé, 2014; Bonami and Beniamine, 2015)

# Alignment strategies in previous work

- ▶ Transducer intersection (Ahlberg, Forsberg, and Hulden, 2014)
- ▶ Hard coded linguistically motivated alignment
  - ▶ Sims (2015): Suffixation
  - ▶ Albright and Hayes (2002): A single change, with a bias:  
Suffixation > Prefixation > Stem-internal alternation (ablaut/inflection)
  - ▶ Bonami and Boyé (2014) : No stem-internal alternation, with a bias:  
Suffixation > Prefixation > Circumfixation
  - ▶ Bonami and Beniamine (2015): Suffixation with a potential stem-internal alternation
- ▶ Alignment that minimizes edit distance:
  - ▶ Durrett and DeNero, 2013 : weighted according to cross-lexeme comparisons.
  - ▶ Albright and Hayes, 2006 : weighted according to phonological similarity.

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# General strategy

Strategy in 3 steps:

1. Align forms locally to find patterns.
2. Generalize by merging patterns with the same structural alternation.
3. Select the patterns which perform best across lexemes.

# Alignment with simple edit distances (I)

- ▶ For any pair of forms, find the set of alignments that minimize a weighted edit distance
- ▶ We define three operations with costs :

PRS.1SG 'I bring'	a	m	ε	n	
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# Alignment with simple edit distances (I)

- ▶ For any pair of forms, find the set of alignments that minimize a weighted edit distance
- ▶ We define three operations with costs :
  - ▶ COPY is free

PRS.1SG 'I bring'	a	m	ε	n	
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Operation	C	C		C	
Cost	0	0		0	

# Alignment with simple edit distances (I)

- ▶ For any pair of forms, find the set of alignments that minimize a weighted edit distance
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  - ▶ INSERTION/DELETION costs 1

PRS.1SG 'I bring'	a	m	ε	n	
PRS.2PL 'you bring'	a	m	∅	n	e
Operation	C	C		C	I
Cost	0	0		0	1

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- ▶ For any pair of forms, find the set of alignments that minimize a weighted edit distance
- ▶ We define three operations with costs :
  - ▶ COPY is free
  - ▶ INSERTION/DELETION costs 1
  - ▶ SUBSTITUTION costs 2 (equivalent to insert + delete)

PRS.1SG 'I bring'	a	m	ε	n	
PRS.2PL 'you bring'	a	m	ø	n	e
Operation	C	C	S	C	I
Cost	0	0	2	0	1

## From alignment to patterns

- Find all best alignments and deduce alternation patterns (back to an imaginary language):

	Alignment	Distance	Pattern
	b a b a		
(i) Prefix	_ _ b a	2	$\epsilon \rightleftharpoons ba / \_ba$
(ii) Suffix	b a _ _	2	$\epsilon \rightleftharpoons ba / ba\_$
(iii) Infix	b _ _ a	2	$\epsilon \rightleftharpoons ab / b\_a$

- List the set of patterns for each alternation:

SG	PL	Patterns
ba	<b>b</b> aba	$\{\epsilon \rightleftharpoons ba / \_ba, \epsilon \rightleftharpoons ba / ba\_, \epsilon \rightleftharpoons ab / b\_a\}$
to	<b>t</b> abo	$\{\epsilon \rightleftharpoons ab / t\_o\}$
ri	<b>r</b> abi	$\{\epsilon \rightleftharpoons ab / r\_i\}$
su	<b>s</b> abu	$\{\epsilon \rightleftharpoons ab / s\_u\}$

# Merging patterns generalizes the context

- ▶ Fuse patterns with identical structural alternations
- ▶ This step attempts to find phonological restrictions on contexts by generalizing sets of phonemes to their smallest natural classes.

$$\left. \begin{array}{l} \epsilon \rightleftharpoons ab / b\_a \\ \epsilon \rightleftharpoons ab / t\_o \\ \epsilon \rightleftharpoons ab / r\_i \\ \epsilon \rightleftharpoons ab / s\_u \end{array} \right\} \epsilon \rightleftharpoons ab / C\_V$$

# Merging patterns captures phonological operations

- ▶ A structural alternation can be a phonological operation such as lengthening

$$\left. \begin{array}{l} a \rightleftharpoons a: / \dots \\ i \rightleftharpoons i: / \dots \end{array} \right\} [aiu] \rightleftharpoons [a:i:u:] / \dots$$



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A real example (Arabic verbs)

$[aiu]_{na} \rightleftharpoons [a:i:u:]_i$

$imp.act.2p.f \rightleftharpoons imp.act.2s.f$

## Merging patterns captures phonological operations

- ▶ A structural alternation can be a phonological operation such as lengthening
- ▶ However, this only works if there is a true phonological naturality

Irish: broadening and attenuation, from Carnie, 2008

(25)

Broadening

Spelling	Sound change	Example
<i>ói</i> → <i>ó</i>	[o:] (no change)	<i>cóir</i> → <i>córa</i>
<i>ei</i> → <i>ea</i>	[ɛ] → [a]	<i>greim</i> → <i>greama</i>
<i>éi</i> → <i>éa</i>	[e:] (no change)	<i>tincéir</i> → <i>tincéara</i>
<i>i</i> → <i>ea</i>	[ɪ] → [a]	<i>rith</i> → <i>reatha</i>
<i>(a)í</i> → <i>(a)ío</i>	[i:] (no change)	<i>feadaíl</i> → <i>feadaíola</i>
<i>ui</i> → <i>o</i>	[ɪ] → [ɔ]	<i>cuid</i> → <i>coda</i>
<i>io</i> → <i>ea</i>	[ɪ] → [a]	<i>lios</i> → <i>leas</i>

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Irish: broadening and attenuation, from Carnie, 2008

(24)

Attenuation

Spelling	Sound change	Example
<i>ea</i> → <i>i</i>	[a] → [ɪ]	<i>fear</i> → <i>fir</i>
<i>ea</i> → <i>ei</i>	[a] → [ɛ]	<i>sceach</i> → <i>sceiche</i>
<i>éa</i> → <i>éi</i>	[e:] (no change)	<i>éan</i> → <i>éin</i>
<i>ia</i> → <i>éi</i>	[iə] → [e:]	<i>iasc</i> → <i>éisc</i>
<i>ío</i> → <i>í</i>	[i:] (no change)	<i>íol</i> → <i>íl</i>
<i>io</i> → <i>i</i>	[i:]/[ɪ]/[ʊ] → [ɪ]	<i>fionn</i> → <i>finn</i>
<i>iu</i> → <i>i</i>	[ʊ] → [ɪ]	<i>fliuch</i> → <i>fliche</i>

## Scoring and choosing

- ▶ Score patterns using the harmonic mean of their coverage and accuracy

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$$\frac{\text{lexemes to which the pattern is applicable}}{\text{total lexemes}}$$

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- ▶ accuracy: proportion of candidates actually instantiating the pattern.

$$\frac{\text{lexemes derived correctly}}{\text{lexemes to which the pattern is applicable}}$$

- ▶ For each lexeme, decide on the pattern with the highest score

	Alignment	Pattern	Coverage	Accuracy	Score
(i) Prefix	b a b a _ _ b a	$\epsilon \Rightarrow$ ba / _ba	1/4	1/1	0.4
(ii) Suffix	b a _ _ b a _ _	$\epsilon \Rightarrow$ ba / ba_	1/4	1/1	0.4
(iii) Infix	b _ _ a b _ _ a	$\epsilon \Rightarrow$ ab / C_V	4/4	4/4	1.0

# More subtle alignments

- Simple edit distances are not always enough to choose the best alignment :

NOM.SG 'chihuahua' (čivav)	t	ʃ	ɪ	v	a	f		(Czech)
GEN.PL 'chihuahua' (čivava)	t	ʃ	ɪ	v	a	v	a	
Operation	C	C	C	C	C	S	I	
Cost	0	0	0	0	0	2	1	=3



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Operation	C	C	C	C	I	I	C	I	
Cost	0	0	0	0	1	1	0	1	=3

# Alignment with phonologically weighted edit distances

- ▶ We use phonological similarity as a weight (Frisch, Pierrehumbert, and Broe, 2004; Albright and Hayes, 2003)
  - ▶ SUBSTITUTION of segments costs  $1 - \text{phonological similarity}$ .

$$\text{sim}(a, b) = \frac{|\text{classes}(a) \cap \text{classes}(b)|}{|\text{classes}(a) \cup \text{classes}(b)|}$$

Copy is a substitution and costs 0, as  $\text{sim}(a, a) = 1$ .

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t	ʃ	ɪ	v	a	f			t	ʃ	ɪ	v			a	f
t	ʃ	ɪ	v	a	v	a		t	ʃ	ɪ	v	a	v	a	
S	S	S	S	S	S	S		S	S	S	S			S	
0	0	0	0	0	0	0.1		0	0	0	0			0	

Czech, ‘chihuahua’, NOM.SG čivav  $\rightleftharpoons$  GEN.PL čivava

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  - ▶ INSERTION is a parameter (here 0.5)

t	ʃ	ɪ	v	a	f			t	ʃ	ɪ	v			a	f
t	ʃ	ɪ	v	a	v	a		t	ʃ	ɪ	v	a	v	a	
S	S	S	S	S	S	I		S	S	S	S	I	I	S	I
0	0	0	0	0	0.1	0.5		0	0	0	0	0.5	0.5	0	0.5

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t	ʃ	ɪ	v	a	f			t	ʃ	ɪ	v			a	f
t	ʃ	ɪ	v	a	v	a		t	ʃ	ɪ	v	a	v	a	
S	S	S	S	S	S	I		S	S	S	S	I	I	S	I
0	0	0	0	0	0.1	0.5		0	0	0	0	0.5	0.5	0	0.5
=0.6							=1.5								

Czech, 'chihuahua', NOM.SG čivav  $\rightleftharpoons$  GEN.PL čivava

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# Evaluation: Qualitative

The algorithm does infer transfixation patterns

$$\_a\_a \rightleftharpoons ju\_i\_u / \_X + \_C\_$$

ka:taba:  
PFV 'he wrote'

juka:tibu  
IPF 'he writes'

$$n\_a\_1^2 \rightleftharpoons \_u\_0^1 / \_k\_0^0 [+con, -lat, -nas] V\_X\_?$$

nka<sup>0</sup>ki<sup>1</sup>tɛ<sup>2</sup>?  
CPL 'she/he broke'

ku<sup>0</sup>ki<sup>0</sup>tɛ<sup>1</sup>?  
POT 'she/he will break'

# Evaluation: Quantitative (I)

- ▶ Can the patterns be used for prediction ?
- ▶ 10-fold cross-validation
- ▶ Training :
  - ▶ Learn patterns on 90% of pairs.
  - ▶ Define a form's **class** as the set of patterns applicable to it.
  - ▶ Compute probabilities of the form  $P(\text{pattern}|\text{class})$ .
- ▶ Test :
  - ▶ Look at single forms for the other 10%.
  - ▶ Find the class for each form.
  - ▶ Predict by maximizing  $P(\text{pattern}|\text{class})$ .
  - ▶ If the class is unknown, or the predicted form is false, fail.



## Evaluation: Quantitative (II)

	Chatino	French	Portuguese	Arabic
train size	351	4689	1791	855
test size	39	521	199	95
Left aligned (suffix)	24	93	92	48
Right aligned (prefix)	54	22	17	0.4
Albright and Hayes, 2002	55	93	94	49
Simple edit distances	58	93	-	87
Similarity based edit distances	58	93	93	87

# Conclusion

- ▶ We presented a fully implemented algorithm to automatise the inference of alternation patterns.
  - ▶ it's open source:  
<http://drehu.linguist.univ-paris-diderot.fr/qumin/>
- ▶ Crucial properties :
  - ▶ Generalizes from fully specified form pairs to alternation patterns, with a phonologically constrained context.
  - ▶ Fast enough to perform on numerous series of pairs (cf. inflection).
  - ▶ Not limited to affixal material.
  - ▶ Does not require prior knowledge on the language's profile.
  - ▶ Optimizes the patterns globally across lexemes as well as locally.
  - ▶ Known patterns can be applied to unknown forms to predict new forms.
- ▶ It has already been used in previous works Beniamine and Bonami, 2016; Bonami and Beniamine, 2016.

Thank You !

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# Evaluation: Quantitative

	Chatino	French	Portuguese	Arabic
train size	351	4689	1791	855
test size	39	521	199	95
Left aligned (suffix)	24	93	92	48
(num. of patterns)	229	26	17	409
Right aligned (prefix)	54	22	17	0.4
(num. of patterns)	54	3612	1467	464
Albright and Hayes, 2002	55	93	94	49
(num. of patterns)	73	26	17	397
Simple edit distances	58	93	-	87
(num. of patterns)	55	26	-	22
Similarity based edit distances	58	93	93	87
(num. of patterns)	54	26	16	23

# A few patterns for English verbs

lexeme	PAST $\rightleftharpoons$ PPART	PAST $\rightleftharpoons$ PRES3S	PRES3S $\rightleftharpoons$ PRESNOT3S	PPART $\rightleftharpoons$ PRESNOT3S	PPART $\rightleftharpoons$ PRESPART
DRIVE	XəvC $\rightleftharpoons$ XiCŋ	XəvC_ $\rightleftharpoons$ XaiCz	Xz $\rightleftharpoons$ X	XiCŋ $\rightleftharpoons$ XaiC	XiCŋ $\rightleftharpoons$ XaiC1ŋ
RIDE	XəvC $\rightleftharpoons$ XiCŋ	XəvC $\rightleftharpoons$ XaiCz	Xz $\rightleftharpoons$ X	XiCŋ $\rightleftharpoons$ XaiC	XiCŋ $\rightleftharpoons$ XaiC1ŋ
BITE	X $\rightleftharpoons$ Xŋ	XiC $\rightleftharpoons$ XaiCs	Xs $\rightleftharpoons$ X	XiCŋ $\rightleftharpoons$ XaiC	XiCŋ $\rightleftharpoons$ XaiC1ŋ
FORGET	X $\rightleftharpoons$ Xŋ	XɔC $\rightleftharpoons$ XɛCs	Xs $\rightleftharpoons$ X	XɔCŋ $\rightleftharpoons$ XɛC	XɔCŋ $\rightleftharpoons$ XɛC1ŋ

lexeme	PAST $\rightleftharpoons$ PRESNOT3S	PPART $\rightleftharpoons$ PRES3S	PRESNOT3S $\rightleftharpoons$ PRESPART	PAST $\rightleftharpoons$ PRESPART	PRES3S $\rightleftharpoons$ PRESPART
DRIVE	XəvC $\rightleftharpoons$ XaiC	XiCŋ $\rightleftharpoons$ XaiCz	X $\rightleftharpoons$ Xiŋ	XəvC $\rightleftharpoons$ XaiC1ŋ	Xz $\rightleftharpoons$ Xiŋ
RIDE	Xəv $\rightleftharpoons$ Xai	XiCŋ $\rightleftharpoons$ XaiCz	X $\rightleftharpoons$ Xiŋ	XəvC $\rightleftharpoons$ XaiC1ŋ	Xz $\rightleftharpoons$ Xiŋ
BITE	XiC $\rightleftharpoons$ XaiC	XiCŋ $\rightleftharpoons$ XaiCs	X $\rightleftharpoons$ Xiŋ	XVt $\rightleftharpoons$ XaitVŋ	Xs $\rightleftharpoons$ Xiŋ
FORGET	XɔC $\rightleftharpoons$ XɛC	XɔCŋ $\rightleftharpoons$ XɛCs	X $\rightleftharpoons$ Xiŋ	XɔC $\rightleftharpoons$ XɛC1ŋ	Xs $\rightleftharpoons$ Xiŋ