Polychrome tree grammars (PTGs): a formal presentation.

Marcel CORI and Jean-Marie MARANDIN

Université Paris 7 and CNRS URA 1028, Case 7003, 2 place Jussieu, 75251 Paris Cedex 05, France Email: mco@ccr.jussieu.fr

0. INTRODUCTION

The motivation of Polychrome Tree Grammars (PTGs) is twofold:

- (i) to give an autonomous representation of the syntactic positions underlining NL utterances,
- (ii) to give a monostratal account of extraclausal constituents (ECs) on the syntactic level.

We introduce branch colours in the definition of what is a tree in order to cater for (i); as for (ii), we redefine the tree-building operation rather than introducing "discontinuous trees" (McCawley 1982, 1987, Huck & Ojeda 1987).

PTGs are conceived as a purely syntactic formalism in an autolexical grammar¹. In this paper, we restrict our study to the syntactic level which we conceive as configurations of unlabelled positions (§1) and focus on the mathematical definitions: polychrome trees (§2), tree composition (§3), tree grammars (§4). We show in §5 how context-sensitivity (CS) is obtained without CS-rules or feature structures.²

1. MOTIVATION

1.1. Positions and constituents

It is commonly accepted that a position such as COMP can be occupied by constituents of different categories. It is illustrated in (1) below:

(1) a. La maison [$_{s'}$ [$_{comp}$ ($_{n''}$ que)] [$_{s}$ tu vois]] b. La maison [$_{s'}$ [$_{comp}$ ($_{p''}$ dans laquelle)] [$_{s}$ tu vis]] c. La maison [$_{s'}$ [$_{comp}$ ($_{p''}$ sur le toit de laquelle)] [$_{s}$ il n'y a pas de girouette]] d. Je me demande [$_{s'}$ [$_{comp}$ ($_{a''}$ quel)] [$_{s}$ sera son destin]] 3

The basic content of X-bar theory is that the kernel position of a syntactic structure can be occupied by a constituent of only one category; in its most popular version, this category is identical to the category of the structure; e.g. the kernel position of N' is occupied only by a constituent of category N and a constituent of category N can occupy only the kernel

¹ The autolexical framework is defined in Sadock (1991). The conception of syntax we refer to is *la syntaxe positionnelle* (positional syntax; Milner 1989). We use level and stratum in the sense defined by Ladusaw (1988).

² For further argumentation from a linguistic point of view, see Cori & Marandin (forthcoming).
3 Gloss: a The bouse which you see: b. The bouse in which you live; a The bouse on the re-

³ Gloss: a. The house which you see; b. The house in which you live; c. The house on the roof of which there is no weathercock; d. I wonder what will be his fate.

positions of N^{'4}.

This claim is not verified in languages such as French or English. Kerleroux (1990,1991) shows that the adjectives and verbs occurring in the italicized phrases (2.a-b) below have undergone no morphological operation of conversion (non-affixal derivation from A or V to N) and that the phrases are NPs; Marandin (forthcoming) shows that there is no empty N in NPs with anaphoric interpretation such as the italicized one in (2.c).

(2) a. [np [det Le] [v manger cru]] aurait des vertus thérapeutiques.
b. Il est d' [np [det un] [a sentimental]]!
c. Parmi ses remarques, je préfère [np [det les] [a grammaticales]].5

In other words, adjectives and nouns may occur in the kernel position of the French NP. Pullum (1991) reaches a similar conclusion in the analysis of nominal gerund phrase (illustrated in (3) below): NPs can have heads that are verb phrases (VP); they are generated by the rule (3.b) in a GKPS framework (Gazdar et al., 1985).

(3) a. Your having broken the record was a surprise. b. N (bar:2) -->(N (BAR:2, POSS: +)), H (VFORM: prp)

The point is that these "heterocategorial" heads (as Pullum calls them) are indeed heads of NP, i.e. behave as heads irrespective of their categorial status; in the same way, constituents occurring in COMP enter the same range of relations independently of their categorial status.

We want to represent the positions without recourse to the categorial status of the constituents which occupy them: straight categories such as N, N" or NP, ad hoc categories like COMP or variables such as H in the GKPS framework.

1.2. Extraclausal constituents on the syntactic level

We consider only one case for the sake of illustration: ECs interpolated into VP to the right of a finite verb (either between the auxiliary and the past participle or to the right of the main verb in a simple tense). This is illustrated in (4):

- (4) a. Paul a, tu le sais, payé la note dimanche matin.
 - b. Paul a, lui, payé la note dimanche matin.

c. Paul paiera, le con, la note dimanche.

d. *Paul paiera, le cousin de Jean, la note dimanche.6

Such ECs are typically "bracketted off" from the clause by pauses; we leave this intonational feature aside. Henceforth we shall call the interpolation site IRV (interpolation to the right of V).

Three features of the structure must be emphasized:

- Selection of the EC: not any constituent may occur in IRV. APs and subordinate or relative clauses are forbidden in IRV. Among NPs, only non-referentially autonomous NPs are possible: hence the possibility of pronouns (4.b) or qualitative NPs (4.c)⁷, and the agrammaticality of fully referential NPs (4.d). Among Ss, only independent Ss (4.a) are

⁴ See Kornaï & Pullum (1990: 42)

⁵ Gloss: a. The eating raw would have therapeutical virtues; b. he's so sentimental; c. amongst his remarks, I prefer the grammatical ones.

⁶ Gloss: a. Paul has paid, as you know, the bill on Sunday morning; b. Paul has him, paid the bill on Sunday morning; c. Paul will pay, the damn fool, the bill on Sunday; d. Paul will pay, Jean's cousin, the bill on Sunday.

⁷ See Milner (1979) for the analysis of qualitative NP.

possible. PPs and Advs must be sentence-oriented.

- Constraint on the interpretation. ECs may serve a variety of pragmatic functions. These functions concern (i) the "management" of the interaction, (ii) comments on the content of the clause (Dik, 1989: 265). In IRV, ECs cannot serve the management function: address terms are excluded from IRV. Typical introducers of free indirect style dit-il, répondit-il, pensait-il (he says, he answered, he thought) are excluded from IRV8. Rather, ECs in IRV are in a topic/comment relation with S; precisely, they must be interpreted as a comment on S. IRV is thus the preferential site for appositive NPs to S:
- (5) a. Paul a, fait extraordinaire, payé la note. b. ?Paul a, un fait extraordinaire parmi d'autres, payé la note.9
- Specificity: the selection and interpretive constraints described above are specific to IRV (other interpolation sites do not exhibit the same selection and the same constraint on the interpretation¹⁰).

These three features qualify IRV as a position in exactly the same sense than the one we use for COMP or the kernel of NP. If the position is localized inside VP, the EC enters a relation outside VP with S as a whole. Hence the apparent conflict in a monodimensionnal framework: the EC does not belong to the VP considered as an interpreted structure 11, IRV belongs to the VP considered as a configuration of positions. It is a clear case of modular mismatch (Sadock, 1991).

We want to generate these configurations of positions without recourse to extra operations (such as transformations).

2. POLYCHROME TREES

2.1. Trees

Let Cat be a set, the set of categories. According to Wall (1972), a tree is defined as a quadruple $A = \langle X, L, D, P \rangle$, where X is a finite set, the set of nodes, L is a function from X into Cat, the labelling function, D is a partial order in X × X (the dominance relation), P is a strict partial order in $X \times X$ (the precedence relation) such that the three following conditions

- $\begin{array}{ll} (i) & \exists \ x \in X \ \forall \ y \in X \ < x,y > \in \ D \\ (ii) & \forall x,y \in X \ \ (< x,y > \in \ P \ \lor < y,x > \in \ P) \Leftrightarrow (< x,y > \not \in \ D \ \land < y,x > \not \in \ D) \\ (iii) & \forall w,x,y,z \in X \ \ < w,x > \in \ P \ \land < w,y > \in \ D \ \land < x,z > \in \ D \Rightarrow < y,z > \in \ P \end{array}$

Condition (i) is the Single Root Condition, condition (ii) is the Exclusivity Condition and condition (iii) is the Nontangling Condition.

2.2. Polychrome Trees

Let p and k be two integers such that $1 \le k \le p$. A quadruple $A = \langle X, L, d, P \rangle$, is a polychrome tree (or a PT) when $d = \langle d_1, d_2, ..., d_p \rangle$ is a sequence of disjoint antireflexive 12

⁸ When dit-il is in IRV, it is glossable by so he says, or according to him and not by he says.

⁹ Gloss: a. Paul has, extraordinary fact, paid the bill; b. Paul has, an extraordinary fact among others, paid the bill.

¹⁰ See Fradin (1988) for data on front and tail insertion site in French.

¹¹ Hence McCawley's observation: EC does not count as a part of the antecedent of a pronoun referring to VP, etc.

¹² A relation R is antireflexive if and only if $\langle x,y \rangle \in R \implies x \neq y$.

relations in X × X such that, if D is the transitive closure of $d_1 \cup d_2 \cup ... \cup d_p$, <X,L,D,P> is a tree (according to Wall) and these supplementary conditions hold:

 $\begin{array}{l} \forall \ x,y,z \in \ X \ < x,y > \in \ d_i \ \land < x,z > \in \ d_j \ \land i < j \Rightarrow < y,z > \in \ P \\ \forall \ x,y,z \in \ X \ < x,y > \in \ d_i \ \land < x,z > \in \ d_i \ \land \ y \neq z \Rightarrow (< y,z > \in \ P \lor < z,y > \in \ P) \end{array}$

 $\begin{array}{ll} (vi) & \forall \ x,y,z \in X \ < x,y > \in \ d_k \ \land < x,z > \in \ d_k \ \Rightarrow y = z \\ (vii) & \forall x,y \in X \ < x,y > \in \ d_i \ \Rightarrow \exists z \in X \ \text{such that} \ < x,z > \in \ d_k \end{array}$

p is the number of colours: $\langle x,y \rangle \in d_i$ means that colour i is attributed to branch $\langle x,y \rangle$. k

represents the kernel position.

Conditions (iv) and (v) are Order Conditions. (iv) says that colours constrain "linear order" and (v) implies that a son of a node cannot dominate another son of the same node. Conditions (vi) and (vii) are Kernel Conditions: there is one and only one node in kernel position of a given node.

2.3. Example of a polychrome tree

We take p = 5, k = 3 and we represent in figure 1 the tree whose branches are described by:

 $d_1 = \{ \langle a,b \rangle \}$ $d_2 = \emptyset$ $d_3 = \{ \langle a,c \rangle, \langle c,d \rangle, \langle d,f \rangle \}$

 $d_4 = \{ \langle d, g \rangle, \langle d, h \rangle \}$

 $d_5 = \{\langle c, e \rangle, \langle d, i \rangle\}$

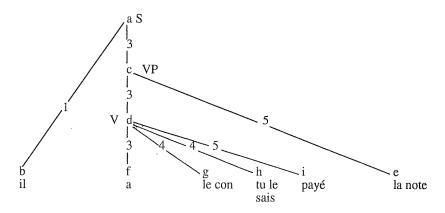


Figure 1.

2.4. Root and leaves

The *root* of a PT is the unique node x_0 such that: $\forall x \in X \ \forall i \in \{1,..,p\} \ \langle x,x_0 \rangle \notin d_i$

We write $Root(A) = x_0$

A *leaf* of a PT is a node z such that:

3. TREE COMPOSITION

3.1. Composability

Let $A' = \langle X', L', d', P' \rangle$ and $A'' = \langle X'', L'', d'', P'' \rangle$ be two polychrome trees such that $X' \cap X'' = \emptyset$; let a be the root of A'' and b a leaf of A'. A' et A'' are *combinable* via b if and only if L'(b) = L''(a). ¹³

Two cases arise. The first one (case 1) is equivalent to ordinary substitution. In the second (case 2) the trees are conflated. Case 2 occurs when b occupies the kernel position of a node labelled with the same category:

 $\exists c \in X' \text{ such that } \langle c, b \rangle \in d_{k'} \text{ and } L'(c) = L'(b)$

3.2. Composition

We take Y and Z as two subsets of X' and X" respectively, the union of which makes up the set of nodes of the combined tree. q is the node of A' which acts as a "hinge" in the composition.

$$\begin{array}{lll} (case\ 1) & Y=X' & Z=X"-\{a\} & q=b \\ (case\ 2) & Y=X'-\{b\} & Z=X"-\{a\} & q=c \end{array}$$

The *combined* polychrome tree via b of A' and A" is the unique PT $A = \langle X, L, d, P \rangle$ (written [A',b,A"]) verifying:

```
 \begin{array}{lll} (i) & X = Y \cup Z \\ (ii.1) & \forall \ x \in Y & L(x) = L'(x) \\ (ii.2) & \forall \ x \in Z & L(x) = L''(x) \\ (iii.1) & \forall \ x \in Y - \{q\} \ \forall \ y \in Y \ \forall \ i \in \{1,...,p\} \ < x,y > \in \ d_i \\ (iii.2) & \forall \ x,y \in Z \ \forall \ i \in \{1,...,p\} \ < x,y > \in \ d_i'' \Rightarrow < x,y > \in \ d_i \\ (iii.3) & \forall \ y \in Z \ \forall \ i \in \{1,...,p\} \ < x,y > \in \ d_i'' \Rightarrow < q,y > \in \ d_i \\ (iv.1) & \forall \ x,y \in Y \ < x,y > \in \ P' \Rightarrow < x,y > \in \ P \\ (iv.2) & \forall \ x,y \in Z \ < x,y > \in \ P'' \Rightarrow < x,y > \in \ P \\ \end{array}
```

Conditions (ii), (iii), (iv) ensure that labelling, dominance and precedence relations are retained by the composition. By condition (iii.3), the nodes immediately dominated by the root of A" are "hitched" to the hinge q.

Further conditions deal with case 2:

$$\begin{array}{lll} \text{(iii.4)} & \forall \ y \in Y \ \forall \ i \in \{1,...,k-1,k+1,...p\} \ \ < c,y> \in \ d_i' \Rightarrow < c,y> \in \ d_i \\ \text{(iv.3)} & \forall \ i < k \ \forall \ x \in Y \ \forall \ y \in Z \ \ < c,x> \in \ d_i' \ \land < a,y> \in \ d_i'' \Rightarrow < x,y> \in \ P \\ \text{(iv.4)} & \forall \ i > k \ \forall \ x \in Y \ \forall \ y \in Z \ \ < c,x> \in \ d_i' \ \land < a,y> \in \ d_i'' \Rightarrow < y,x> \in \ P \end{array}$$

Condition (iii.4) ensures that sons of c in A' remain sons of c in A except the kernel position node. Conditions (iv.3) and (iv.4) take care of the interleaving of the branches of A' around the kernel position of A'. In this case, the composition has a *compacting* effect.

We have to prove that there is effectively a unique PT verifying these conditions. It essentially consists in proving the existence of a unique relation P.

In case 1, let $x \in X'$ such that $\langle x,b \rangle \notin D'$. Then $\langle x,b \rangle \in P'$ or $\langle b,x \rangle \in P'$.

We know that \forall $y \in X'' < b, y > \in D$. Thus, if $< x, b > \in P'$ then $< x, b > \in P$ and, because $< x, x > \in D, < x, y > \in P$. If $< b, x > \in P'$ then $< y, x > \in P$.

In case 2 we have

$$\forall$$
 y \in X" - {a} \exists z \in X" - {a} \exists i such that $<$ a,z $>$ \in di" and $<$ z,y $>$ \in D"

¹³ If the condition $X' \cap X''$ is not verified, nodes of A' or A'' can be renamed to meet the condition.

Let $x \in X' - \{b\}$; three cases arise: (a) $\langle x,c \rangle \in D'$; then $\forall y \in X'' - \{a\} \langle x,y \rangle \in D$ (b) $\langle c,x \rangle \notin D'$ and $\langle x,c \rangle \notin D'$; it is like in case 1. (c) $\exists j \ (\neq k)$ such that $\langle c,x \rangle \in d_j'$. We have $\langle c,x \rangle \in d_j$ and $\langle c,z \rangle \in d_j$. If i < j then $\langle z,x \rangle \in P$ (because 2.2.iv) and $\langle y,x \rangle \in P$. If i > j k then $\langle x,y \rangle \in P$. If i = j < k then $\langle x,z \rangle \in P$ (because 3.iv.3) and $\langle x,y \rangle \in P$. If i = j > kthen $\langle y, x \rangle \in P$.

We can conclude that in each case, if $x \in Y$, $y \in Z$ and $\langle x,y \rangle \notin D$, either $\langle x,y \rangle \in P$ or $\langle y, x \rangle \in P$ and there is only one manner of determining it.

3.3 Examples of composition

Again we have p = 5 and k = 3. The given examples (figures 2.a and 2.b) show the differences between the two cases. In the first case L'(c) = U and L'(b) = V whilst in the second case L'(c) = L'(b) = U. The combined tree is deeper in the first case.

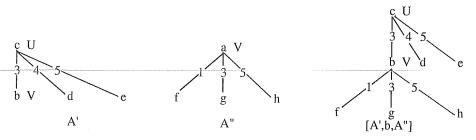


Figure 2.a

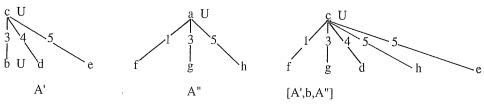


Figure 2.b

3.4. Properties

Let A_1 , A_2 and A_3 be three polychrome trees such that $X_1 \cap X_2 = \emptyset$, $X_2 \cap X_3 = \emptyset$ and

Property 1: If b is a leaf of A_1 , b' a leaf of A_2 , and if A_1 and A_2 are combinable via b and A_2 and A_3 are combinable via b', then: $[[A_1,b,A_2],b',A_3] = [A_1,b,[A_2,b',A_3]]$

Property 2: If b and b' are two leaves of A_1 , and if A_1 and A_2 are combinable via b and A_1 and A_3 are combinable via b', then: $[[A_1,b,A_2],b',A_3] = [[A_1,b',A_3],b,A_2]$

Thus the order of tree composition is not significant.

4. TREE GRAMMARS

4.1. Definitions

A PT is *poor* if and only if:

$$\forall$$
 i \in {1,...,p} \forall x,y,z \in X \langle x,y> \in d_i \land \langle x,z> \in d_i \Rightarrow y = z

A PT is elementary if and only if it is a poor PT such that $\forall \ i,j \in \{1,...,p\} \ \forall \ w,x,y,z \in X \ < x,y > \in \ d_i \ \land < w,z > \in \ d_j \implies x = w$

A Polychrome Tree Grammar (PTG) is given by a finite set of elementary trees: $B = \{A_1, A_2, ..., A_m\}$

This grammar *generates* a set T(B) of PTs:

(i) every A; belongs to T(B);

(ii) if A and A' belong to T(B), then every [A,b,A'] belongs to T(B).

Remark: Using the property 1 in 3.4, (ii) may be replaced by

(ii') if A belongs to T(B) and A' to B, then every [A,b,A'] belongs to T(B).

4.2. A sample grammar for French

In figure 3.a is shown a sample grammar (with p = 5 and k = 3) and in figure 3.b two examples of trees generated by this grammar.¹⁴

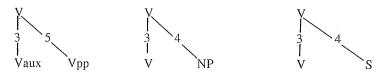


Figure 3.a

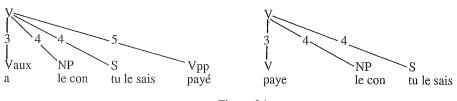


Figure 3.b

Trees in figure 3.b illustrate the second case of composition. They yield the position structure of utterances with EC in IRV; IRV is represented by the position <V,4> in the two last trees of 3.a. Notice that we obtain a (syntactically) unbounded number of ECs in free order, which is precisely what we want to obtain.

¹⁴ The labelling is reduced to the category in this paper. A more complex one, including features, is required to capture the selection restrictions exerted by the positions we mentioned earlier. See Cori & Marandin (forthcoming).

5. CONTEXT-SENSITIVITY

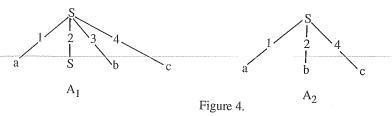
5.1. String language generated by a PTG

We define the word associated with a PT A as being the sequence of leaf labels ordered according to precedence order. So, if Leaves(A) = $z_1...z_s$, we write: Word(A) = $L(z_1)...L(z_s)$

Let V_T be a subset of Cat and S a distinguished element of Cat. We consider the subset $T_T(B)$ of T(B) made of all trees A such that L(Root(A)) = S and $Word(A) \in V_T^*$. The string $SL(B) = \{Word(A) \; ; \; A \in T_T(B)\}$

5.2. An example of a grammar generating aⁿbⁿcⁿ

We take p = 4, k = 2 and the grammar B is made of the two elementary trees represented in figure 4.



Property: The trees of T(B) - $T_T(B)$ are all PTs verifying:

$$\begin{split} \hat{X} &= \{x_0, u_1, ..., u_r, v_1, ..., v_r, w_1, ..., w_r, z\} \\ L(x_0) &= L(z) = S \\ \forall \ i & L(u_i) = a \\ \forall \ i & L(v_i) = b \\ \forall \ i & L(w_i) = c \\ d_1 &= \{; \ 1 \le i \le r \ \} \\ d_2 &= \{\} \\ d_3 &= \{; \ 1 \le i \le r \ \} \\ d_4 &= \{; \ 1 \le i \le r \ \} \end{split}$$

Proof: It is given inductively. The property is true for trees of B. Consider now a PT A verifying the above conditions. We obtain another tree of T(B) - $T_T(B)$ by combining this tree via z with tree A_1 (according to the remark in 4.1). The new tree verifies:

$$\begin{array}{l} X = \{x_0, u, u_1, ..., u_r, v, v_1, ..., v_r, w, w_1, ..., w_r, z'\} \\ L(x_0) = L(z') = S \\ L(u) = a \text{ and } \forall i \ L(u_i) = a \\ L(v) = b \text{ and } \forall i \ L(v_i) = b \\ L(w) = c \text{ and } \forall i \ L(w_i) = c \\ d_1 = \{; 1 \le i \le r \} \cup \{\} \\ d_2 = \{\} \\ d_3 = \{; 1 \le i \le r \} \cup \{\} \\ d_4 = \{; 1 \le i \le r \} \cup \{\} \end{array}$$

Property: The trees of $T_T(B)$ are all PTs verifying:

$$X = \{x_0, u_1, ..., u_r, v_1, ..., v_{r-1}, w_1, ..., w_r, y\}$$

 $L(x_0) = S$

```
∀i
            L(u_i) = a
L(y) = b and \forall i L(v_i) = b
           L(w_i) = c
d_1 = \{ \langle x_0, u_i \rangle ; 1 \le i \le r \}
d_2 = \{\langle x_0, y \rangle\}
d_3 = \{ \langle x_0, v_i \rangle ; 1 \le i \le r-1 \}
d_4 = \{\langle x_0, w_i \rangle ; 1 \le i \le r \}
```

Proof: It is obtained by combining a tree of T(B) - T_T(B) with A₂.

Property: $SL(B) = \{a^nb^nc^n; n > 0\}$

Proof: For each PT A in T_T(B), we have: Leaves(A) = $u_1...u_r$ y $v_1^1...v_{r-1}^rw_1...w_r$, and then $Word(A) = a^rb^rc^r$.

6. CONCLUSION

The autolexical approach requires that the informational patterns of each dimension of NL utterances "be exactly characterized by a generative grammar in the broad sense, that is, a grammar that is explicit and formal and that makes clear and testable predictions" (Sadock, 1991: 5). PTGs appear to be a sound formalism for characterizing the syntactic level. From the empirical point of view, it provides the explicit framework required by the description of the selectional and interpretive properties of syntactic positions. From the formal point of view, we are currently studying the type of context-sensitivity it gives to syntactic representation systems.

REFERENCES

Cori M. & Marandin J.-M., forthcoming, "Grammaires d'arbres polychromes", Paris: TAL. Dik S., 1989, The theory of Functional Grammar, vol I, Dordrecht: Foris Publications. Fradin B., 1988, "Approche des contructions à détachement: la reprise interne", Langue

Gazdar G., Klein E., Pullum G. & Sag I., 1985, Generalized Phrase Structure Grammar, Cambridge MA: Harvard University Press.

Huck G. & Ojeda A., 1987, Discontinuous Constituency, New-York: Academic Press. Kerleroux F., 1990, "Du mode d'existence de l'infinitif substantivé en français contemporain", Cahiers de grammaire 15: 57-99, Toulouse: U. de Toulouse-Le Mirail.

Kerleroux F., 1991, "Il est d'un calme! et d'un élégant! Un phénomène de distorsion (....)", BULAG 17: 79-116, Besançon: Université de Franche-Comté.

Kornaï A. & Pullum G., 1990, "The X-bar theory of phrase structure", Language 66: 24-50. Ladusaw B, 1988, "A proposed distinction between Levels and Strata", Linguistic in the Morning Calm, Seoul: Hanshin Publishing Co. Marandin J.-M., forthcoming, "Les groupes nominaux sans nom en français".

McCawley J., 1982, "Parentheticals and Discontinuous Constituent Structure", Linguistic Inquiry 13: 91-106.

McCawley J., 1987, "Individuation in and of Syntactic Structures", [Baltin et al., Alternative Conceptions of Phrase Structure]: 117-138, Chicago: University of Chicago Press.

Milner J.-C., 1979, De la syntaxe à l'interprétation, Paris: Le Seuil. Milner J.-C., 1989, Introduction à une science du langage, Paris: Le Seuil.

Pullum G., 1991, "English nominal gerund as noun-phases with verb phrase heads", Linguistics 29: 763-799.

Sadock J., 1991, Autolexical Syntax, Chicago: University of Chicago press.

Wall R., 1972, Introduction to Mathematical Linguistics, Englewood Cliffs: Prentice-Hall.